WJEC MATHEMATICS HIGHER TIER QUESTIONS BY TOPIC, NOVEMBER 2016-

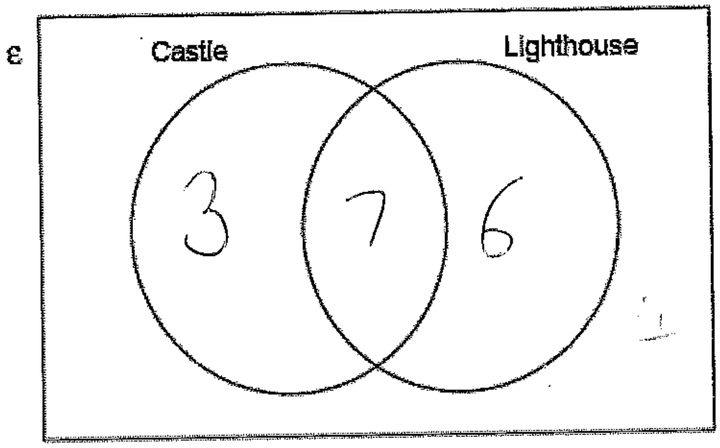
MTIS SOLUTIONS

TOPIC	PAGE nos.
1.Venn Diagrams	2 - 5
2.Transformations R,R,E,T	6 – 13
3. Gradients y = mx + c	14 – 16
4. Relative Frequency	17 – 18
5. Constructions	19 – 21
6. Circle Theorems	22 – 24
7. Trial & Improvement	25 – 27
8. Simultaneous Equations	28 – 30
9. Trigonometry	31 -33
10. Equations U2	34 - 38
11. Equations U1 inc standard form	39 - 49
12. Volume / Surface area	50 -52
13. –ve enlargement	53 - 54
14. Graph Types	55 - 57
15. Trigonometric Graphs	58 – 60
16. Trapezium rule	61
17.Transformation of Graphs f(x)	62 – 64
18. Surds	63 – 65
19. Sine Cosine Rule	66 - 68
20. Simplifying Fractions	69
21. LSF ASF VSF	70
22. Quadratics U2	71 - 72
23. Quadratics U1	73 - 78
24. Proportion	77 - 78
25. Probability inc trees	79 - 85
26. Make x the subject	86
27. Solving/ drawing inequalities	87 – 90
28. Index laws & powers	91 – 93
29. Errors in Measurements, Boundaries	94 - 95
30. Circle Theorems	96 – 101
31. Arcs & Sectors	102 – 103
32. Congruence SSS,RHS,ASA,SAS	104 – 106
33. 3D Trigonometry/Pythagoras	107
34. Dimensions	108

A group of 20 people visited Anglesey for a weekend break.

- 10 of the group visited Beaumaris Castle.
- 13 of the group visited South Stack Lighthouse.
- 4 of the group did not visit either of these places.

(a) Complete the Venn diagram below to show this information. The universal set, ε, contains all of the 20 people in the group. [3]



$20 - 4 = 16$

$13 + 10 - 16 = 7$

(b) One person is chosen at random from the group. What is the probability that this person visited only one of the two places? [2]

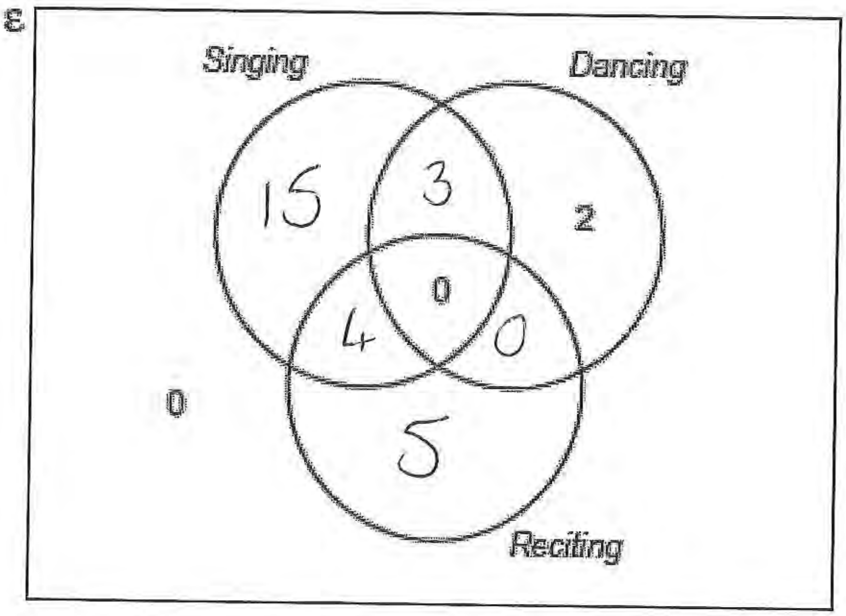
$\frac{9}{20}$

A group of pupils from a school took part in The Urdd National Eisteddfod.
All of them competed in at least one of the following competitions: *Singing*, *Dancing* or *Reciting*.

- 2 of them only took part in a *Dancing* competition.
- 5 only took part in a *Reciting* competition.
- No one took part in both a *Reciting* and a *Dancing* competition.
- 3 took part in both a *Singing* and a *Dancing* competition.
- 9 took part in a *Reciting* competition.
- 22 took part in a *Singing* competition.

The Venn diagram below shows some of the above information.
The universal set, ε, contains all of the pupils in the group.

One of the pupils in the group is chosen at random.
What is the probability that this person only took part in a *Singing* competition? [5]

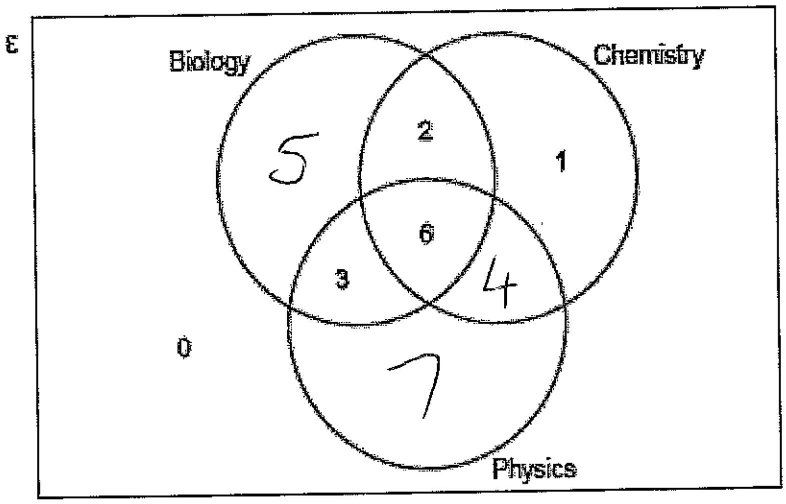


At a college, a total of 28 students study one or more of the science subjects: Biology, Chemistry and Physics.
The 28 students form the universal set, ε.
Some parts of the Venn diagram below have already been completed.

It is also known that:

- 5 students study only Biology
- 13 students study Chemistry

(a) Complete the Venn diagram. [3]



(b) How many students study Biology and Chemistry but not Physics? [1]

2

(c) One of the students is chosen at random.
What is the probability that this student studies Biology? [2]

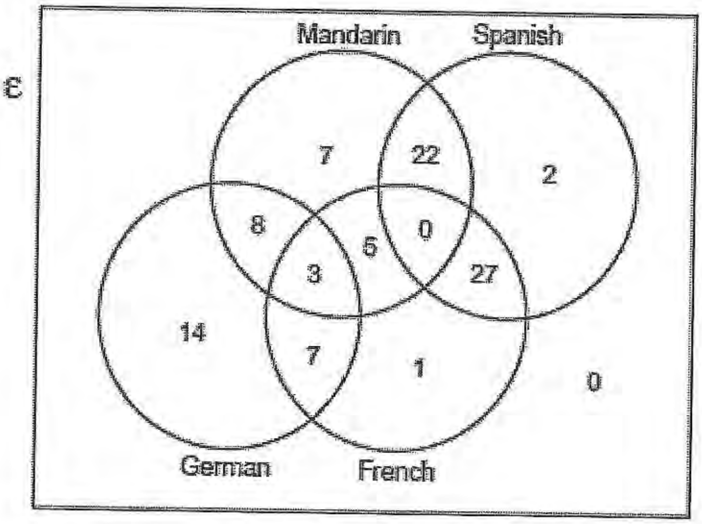
$\frac{16}{28}$

The Headteacher of Ysgol Maes Newydd gave *option forms* to all Year 9 pupils.

The form asked which foreign languages the pupils would like to study in Year 10.

There were 4 languages listed on the form: French, German, Spanish and Mandarin.
The pupils could select as many of the languages as they wished.
All pupils completed and returned the *option form*.

The Headteacher displayed the results in a Venn diagram, as shown below.



(a) How many pupils did not select at least one of the four languages?
Circle your answer. [1]

0 1 3 5 7

(b) How many pupils are there in Year 9?
Circle your answer. [1]

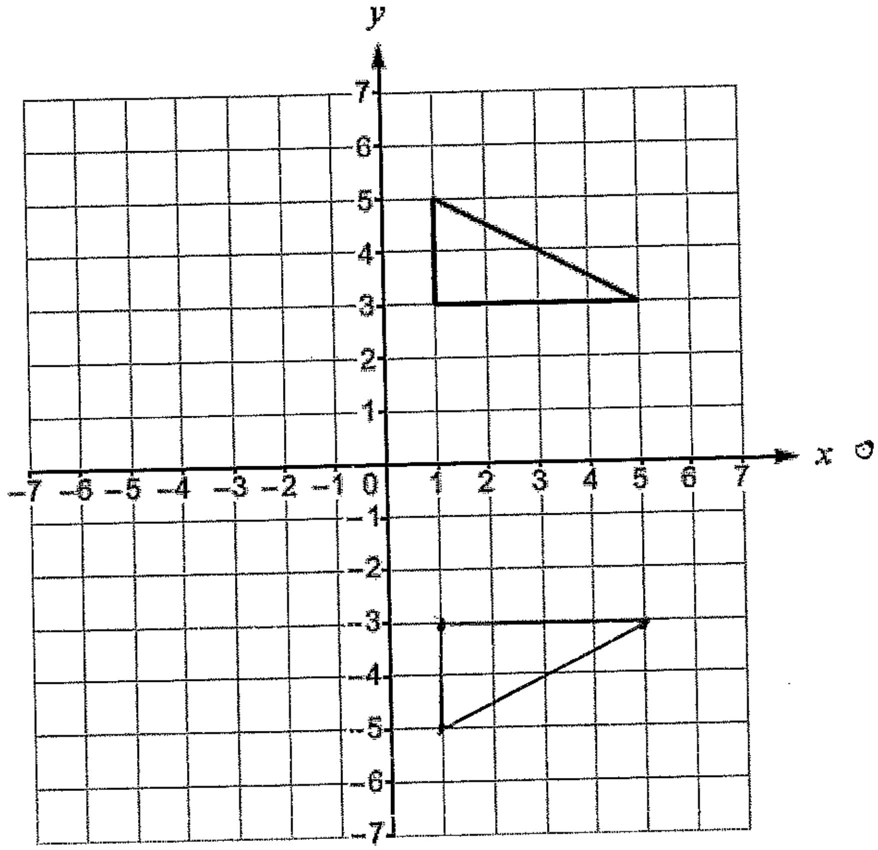
92 94 96 98 100

(c) How many pupils selected only one language? [1]

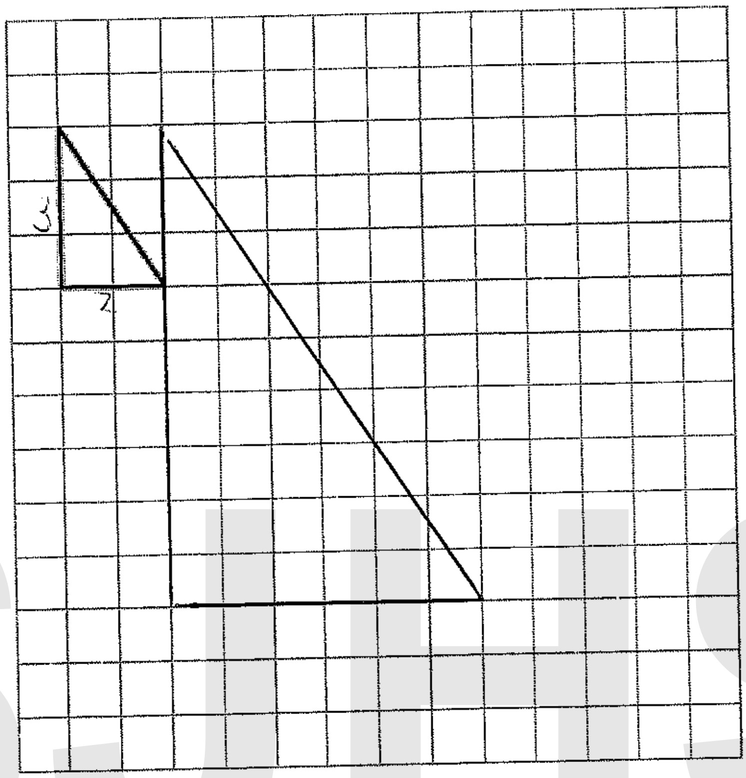
14 24

+1
+7
+2

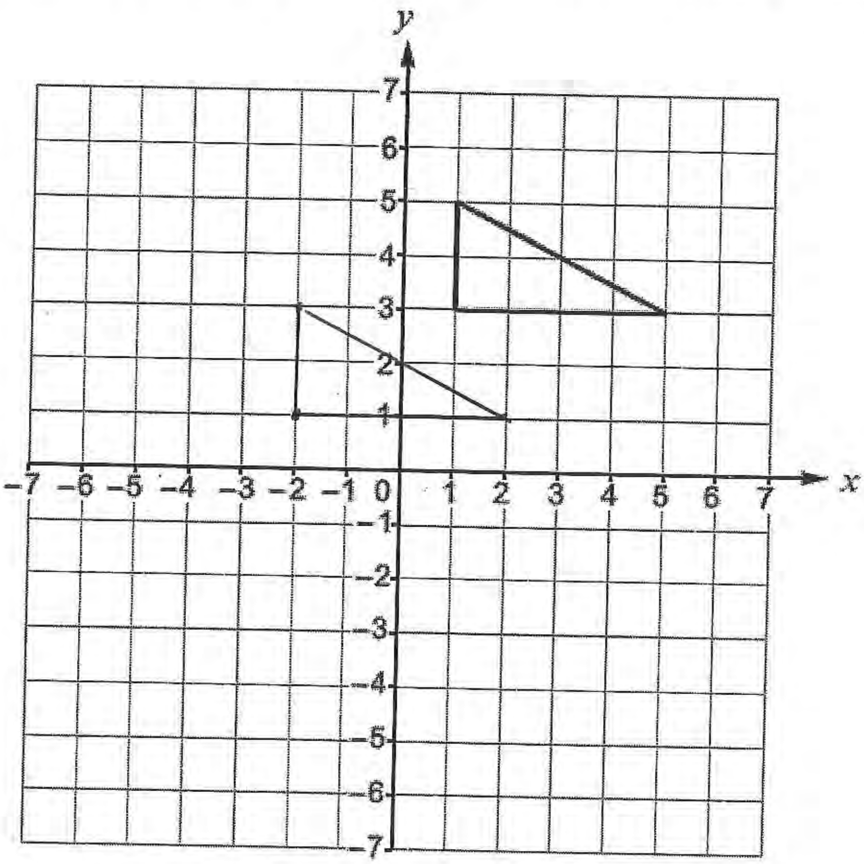
(a) Reflect the triangle below in the x-axis. [1]



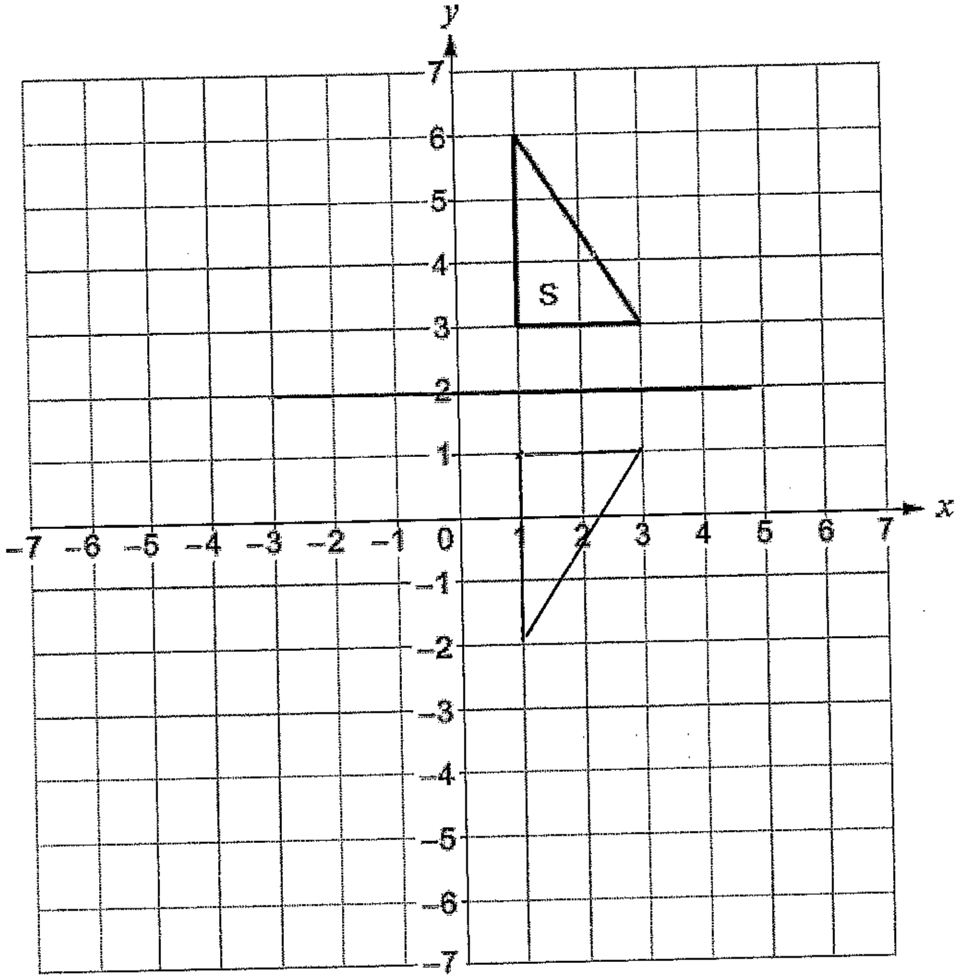
(b) Enlarge the triangle below by a scale factor of 3. [2]



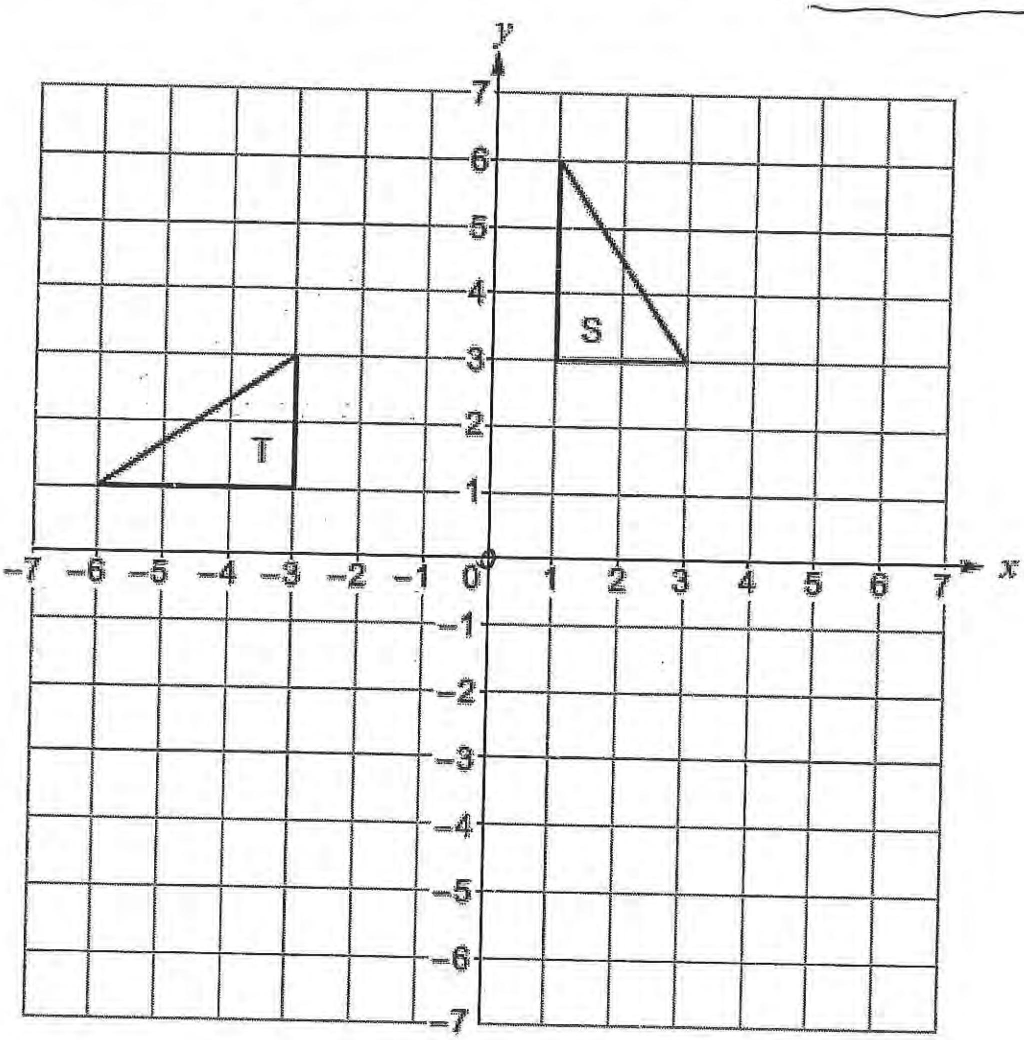
(c) Translate the triangle below 3 squares to the left and 2 squares down. [1]



(a) Reflect the triangle S in the line $y = 2$. [2]

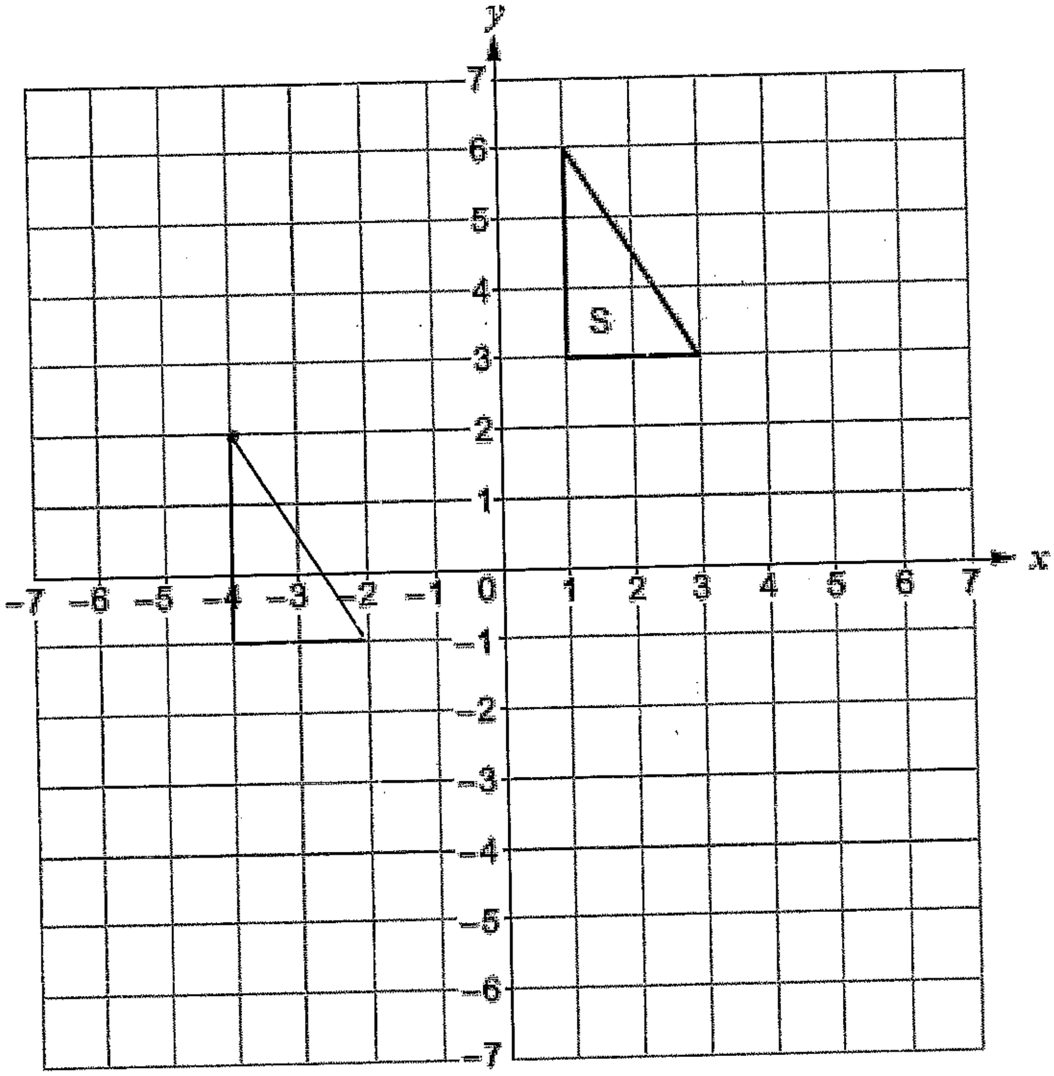


(b) Describe fully a single transformation that transforms triangle S onto triangle T. [3]



Rotate 90° anticlockwise about the point (0,0)

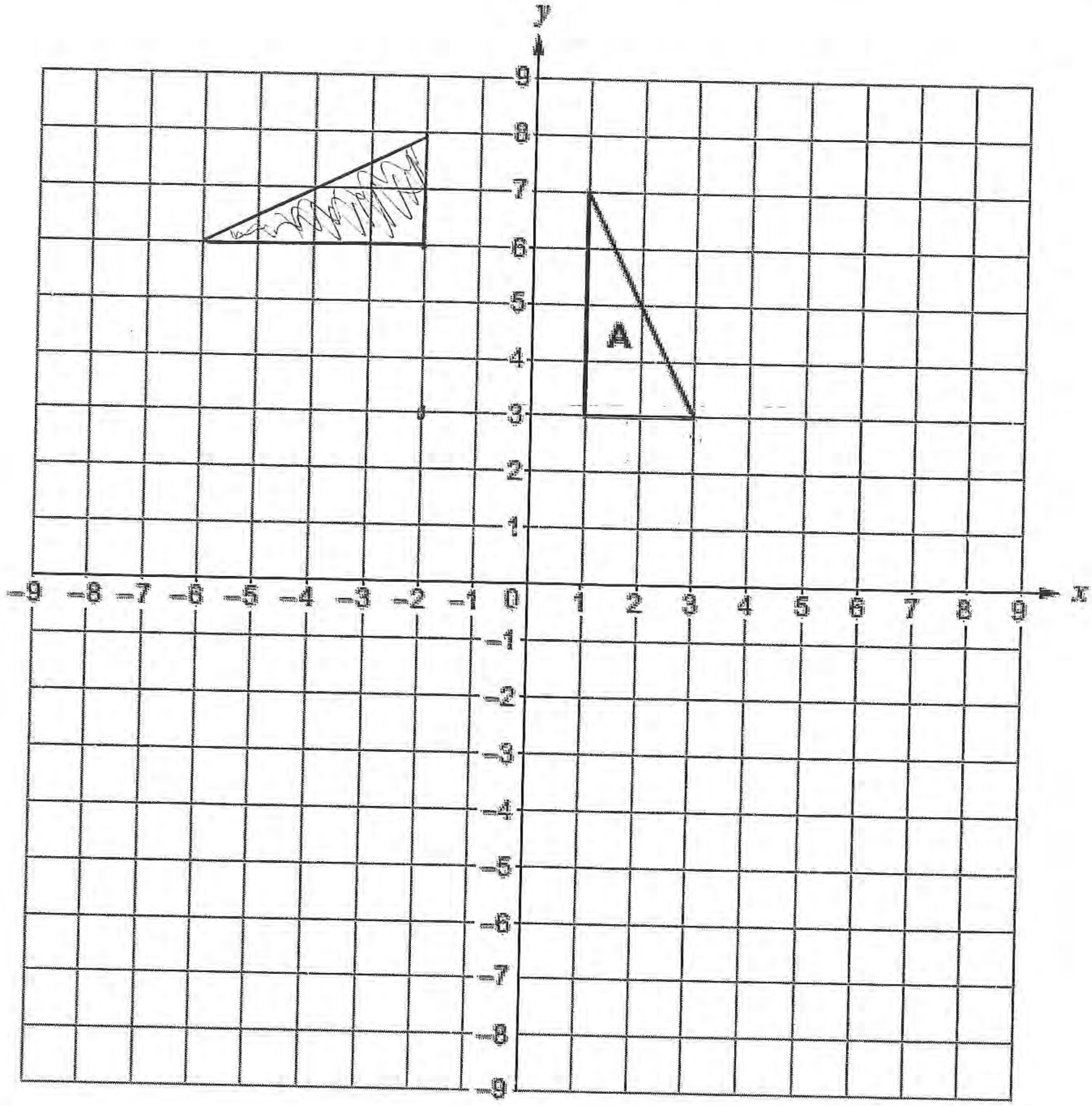
(c) (i) Translate the triangle S using the column vector $\begin{pmatrix} -5 \\ -4 \end{pmatrix}$. [1]



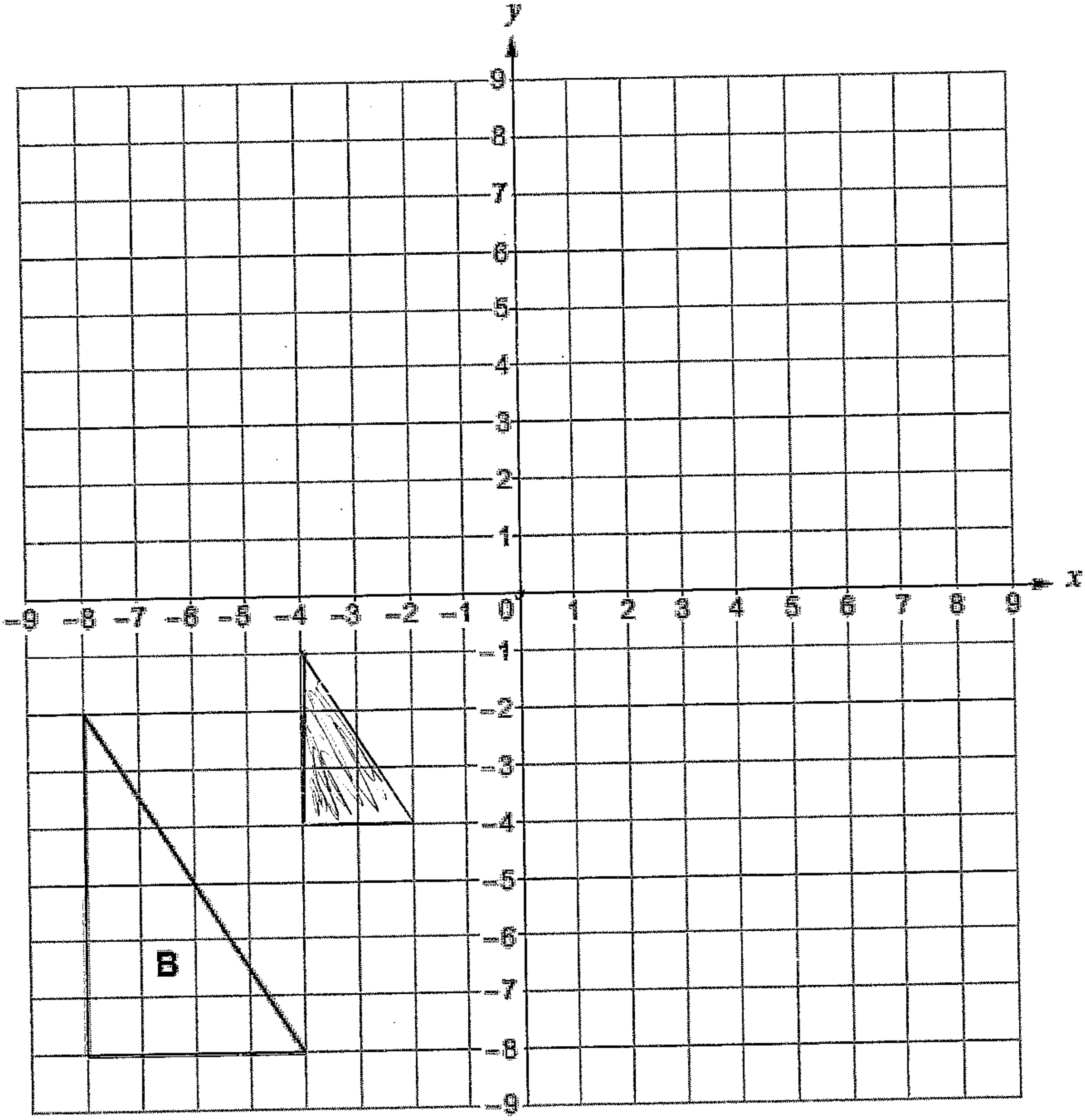
(ii) Write down the column vector that will reverse the translation in part (i). [1]

$\begin{pmatrix} 5 \\ 4 \end{pmatrix}$

(a) Rotate triangle A through 90° anticlockwise, about the point (–2, 3). [2



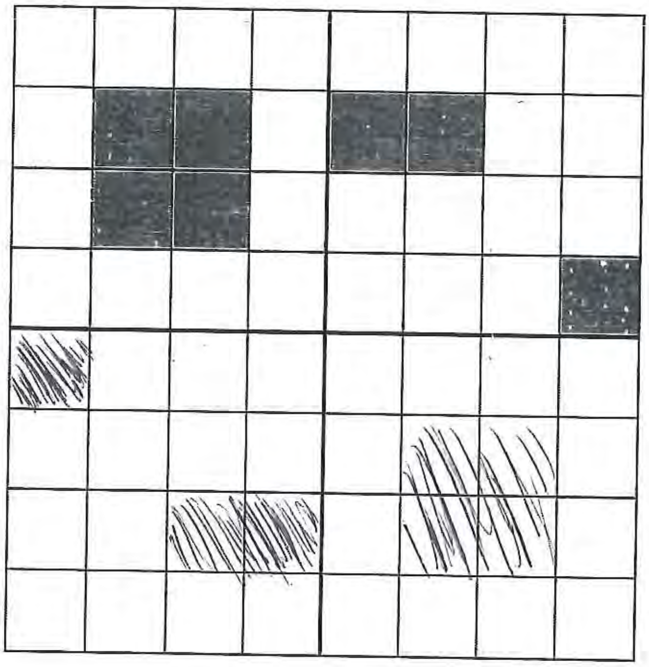
(b) Enlarge triangle B by a scale factor of $\frac{1}{2}$, using (0, 0) as the centre of enlargement. [2]



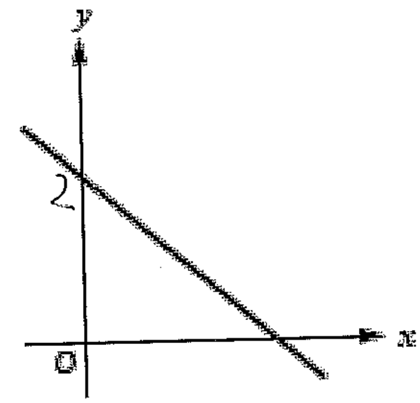
$\leftarrow 8$ $\quad$ $4 \leftarrow$

$\downarrow 2$ $\quad$ $1 \downarrow$

Shade the least number of squares in the lower two quadrants so that the grid has rotational symmetry of order 2.
[3]



(a)



Which one of the following equations could represent the line shown in the graph above? [1]
Circle your answer.

$y = -x - 2$ $\quad$ $y = -x + 2$ $\quad$ $y = x + 2$ $\quad$ $y = x - 2$ $\quad$ $y = -x$.

(b) Which one of the following points lies on the line $2y = 3x + 4$? [1]
Circle your answer.

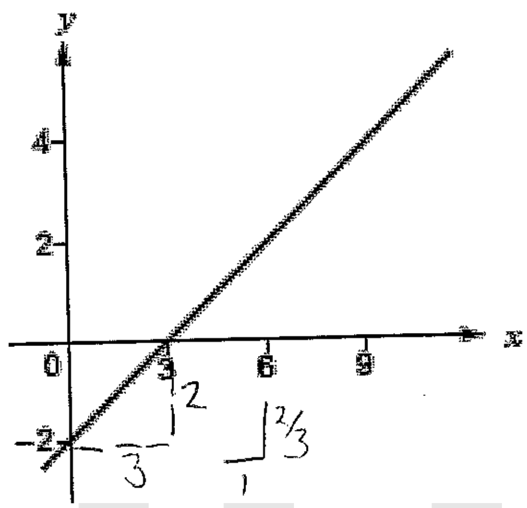
(2, −5) $\quad$ (5, 2) $\quad$ (−2, 5) $\quad$ (2, 5) $\quad$ (−2, −5)

$2x - 5 = 3x - 2 + 4$

$2 \times 5 = 3 \times 2 + 4$ $\quad$ $-10 = -6 + 4$ ✗

$10 = 10$ ✓

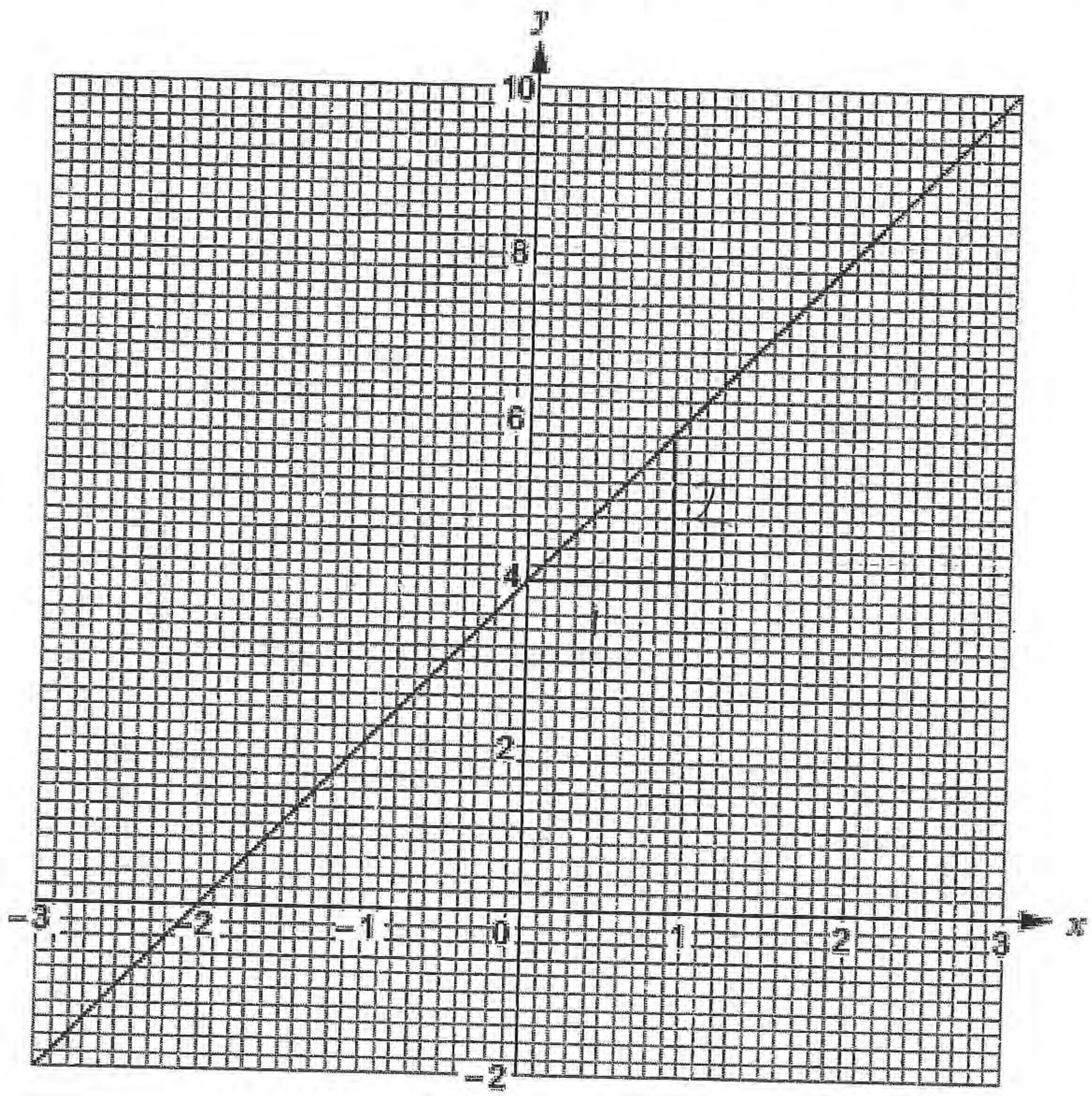
(c)



What is the gradient of the line shown in the graph above? [1]
Circle your answer.

$\frac{3}{2}$ $\quad$ $-\frac{3}{2}$ $\quad$ $\frac{2}{3}$ $\quad$ $-\frac{2}{3}$ $\quad$ -6

(a) The diagram below shows the graph of a straight line for values of x from -3 to 3.



(i) Write down the gradient of the above line. [1

$y = 2x + 4$ 2

(ii) Write down the equation of the line in the form $y = mx + c$, where m and c are wh
numbers. [2

$y = 2x + 4$

(b) Without drawing, show that the line $2y = 5x - 3$ $\times 2$ is parallel to the line $4y = 10x + 7$.
You must show working to support your answer. [2

$4y = 10x - 6$ $\div 4$

$4y = 10x + 7$

Both have same gradiet of

2.5

19. *(a)* Circle the equation of a straight line that is parallel to the line $3y = 2x + 6$. [1]

$3y = 2x + 7$ $\qquad 2y = 3x + 6$ $\qquad 3y = -2x + 6$ $\qquad -3y = 2x + 6$ $\qquad 2y = -3x + 6$

....

....

(b) Circle the equation of a straight line that is perpendicular to the line $y = 5x - 3$. [1]

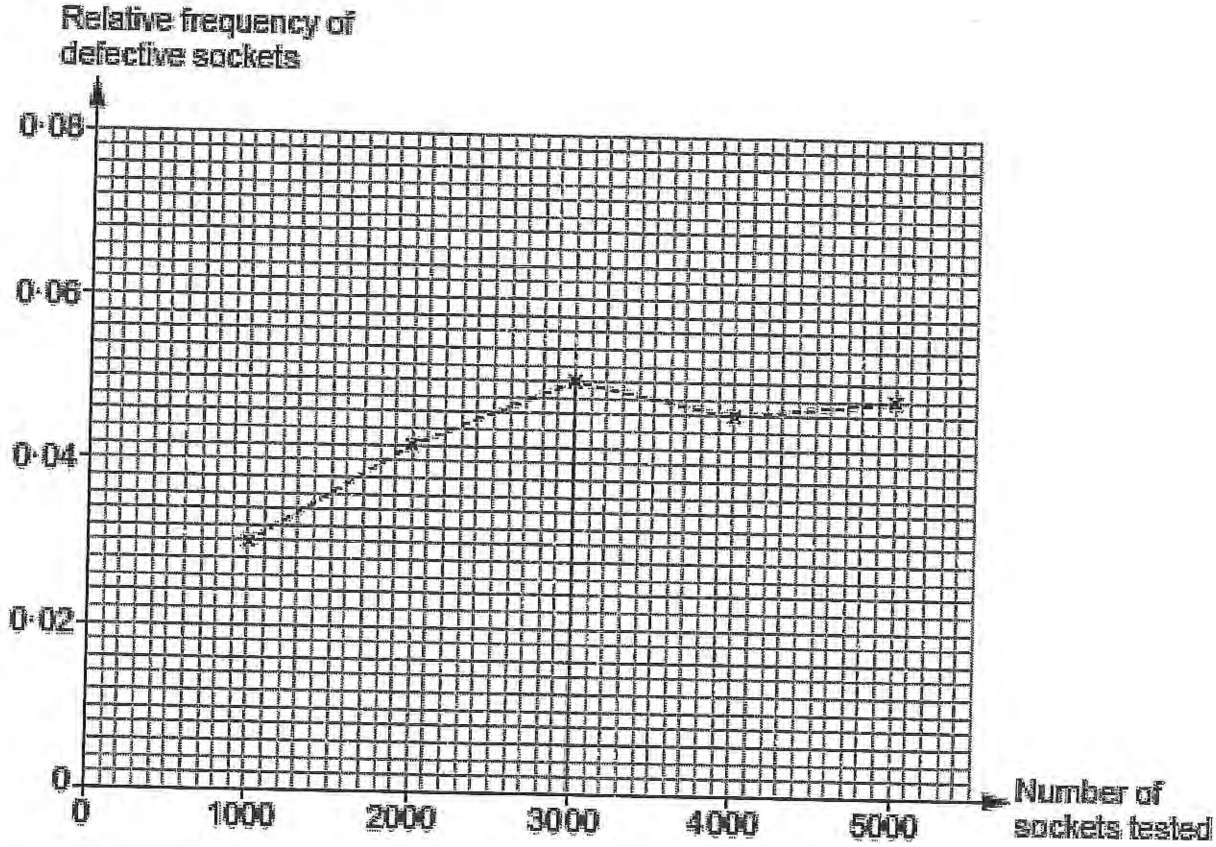
$y = \frac{x}{5} + 3$ $\qquad y = 5x + 3$ $\qquad y = 5x + \frac{1}{3}$ $\qquad y = -5x + 3$ $\qquad y = \frac{-x}{5} + 3$

....

$5 \quad -\frac{1}{5}$

A factory uses a machine to produce electrical sockets.
The manager carries out a survey to investigate the probability of the machine producing a defective socket.

The relative frequency of defective sockets produced was calculated after testing a total of 1000, 2000, 3000, 4000 and 5000 sockets.
The results are plotted on the graph below.



(a) How many of the first 3000 sockets tested were defective? [2]

0.05 x 3000 = 150

(b) Write down the best estimate for the probability that one socket, selected at random, will be defective.
You must give a reason for your choice. [2]

Probability: 0.048

Reason: Most sockets tested so most reliable

A dice is thrown 50 times.
The number shown on the dice is recorded after each throw.
The table below shows the results recorded.

Number shown on dice	1	2	3	4	5	6
Frequency	9	7	8	7	6	13

(a) The relative frequency of throwing a 1 was calculated as $\frac{9}{50} = 0.18$.

What was the relative frequency of throwing a 6? [1]
Give your answer as a decimal.

$\frac{13}{50} = 0.26$

(b) The number 4 was thrown 7 times in the first 50 throws.
Using this fact, calculate how many times you would expect a 4 to be thrown when this dice is thrown 3000 times. [2]

$\frac{7}{50} \times 3000 = 420$

(c) How many times would you expect a 4 to be thrown when a fair dice is thrown 3000 times? [2]

$\frac{1}{6} \times 3000 = 500$

A regular polygon has exterior angles of 45°.

(a) How many sides does this polygon have? [2]

$360 \div 8 = 45 \quad 360 \div 45 = 8$

8 sides

(b) Each side of this regular polygon is 7 cm.
A sketch of two sides, AB and BC, of the polygon is shown below.

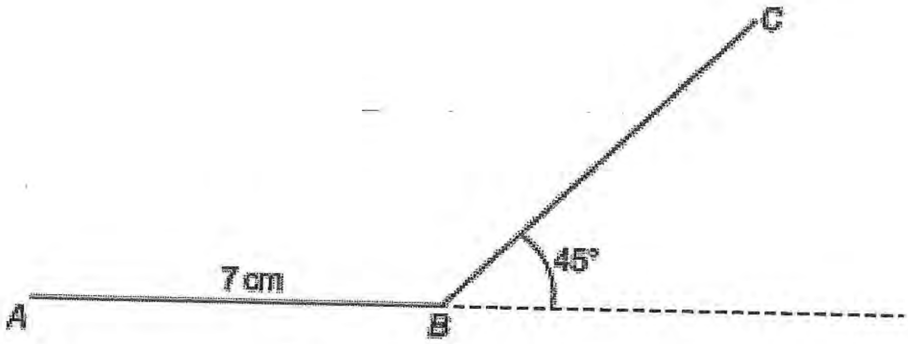
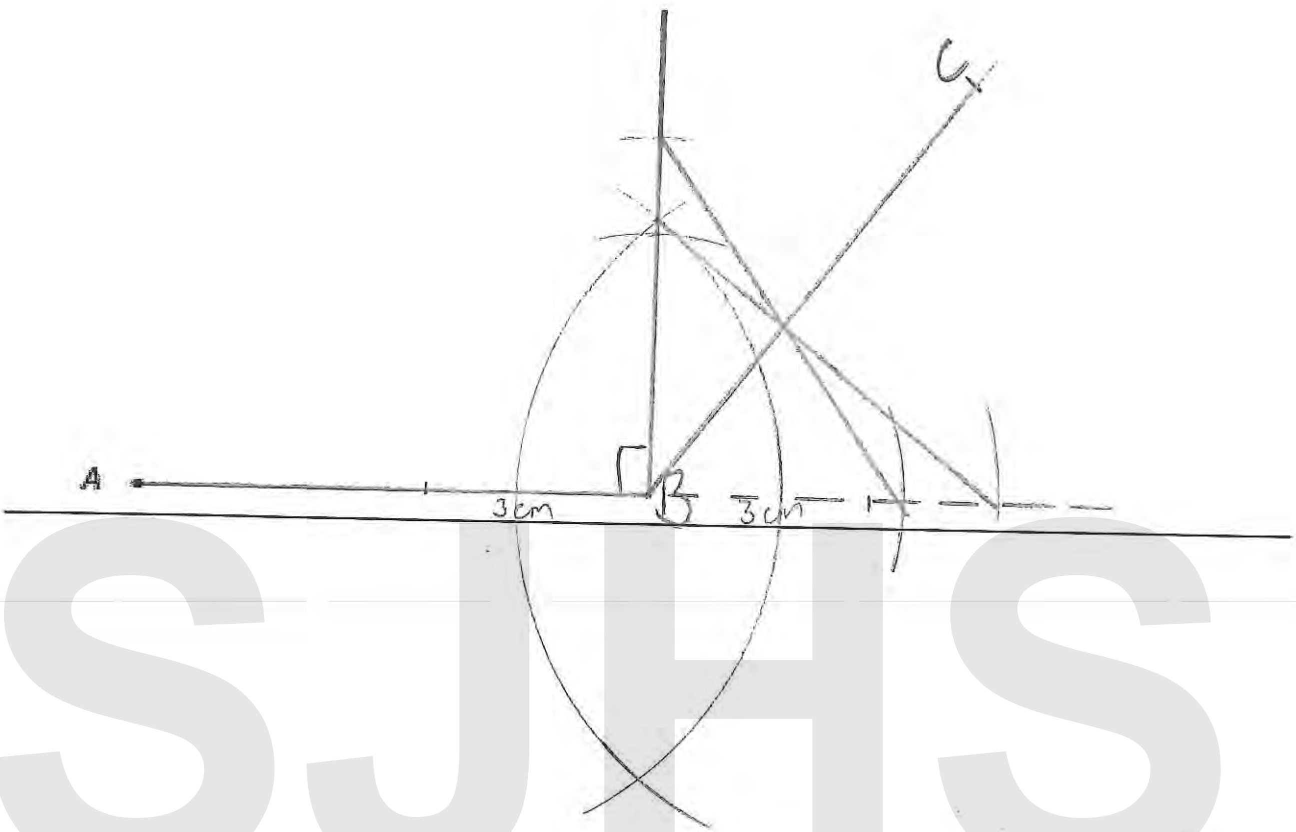
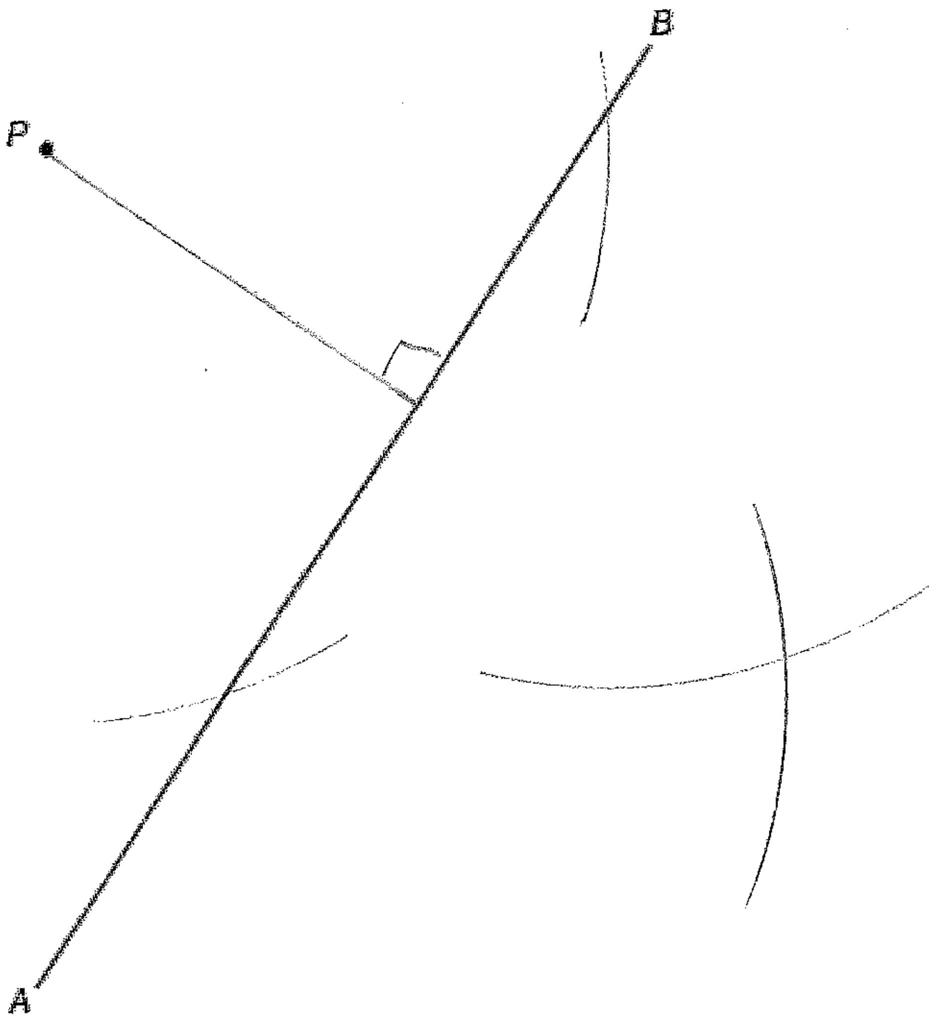


Diagram not drawn to scale

Using only a ruler and a pair of compasses, construct an accurate drawing that shows these two sides of the polygon.
The point A has been given.
You must show your construction arcs. [4]

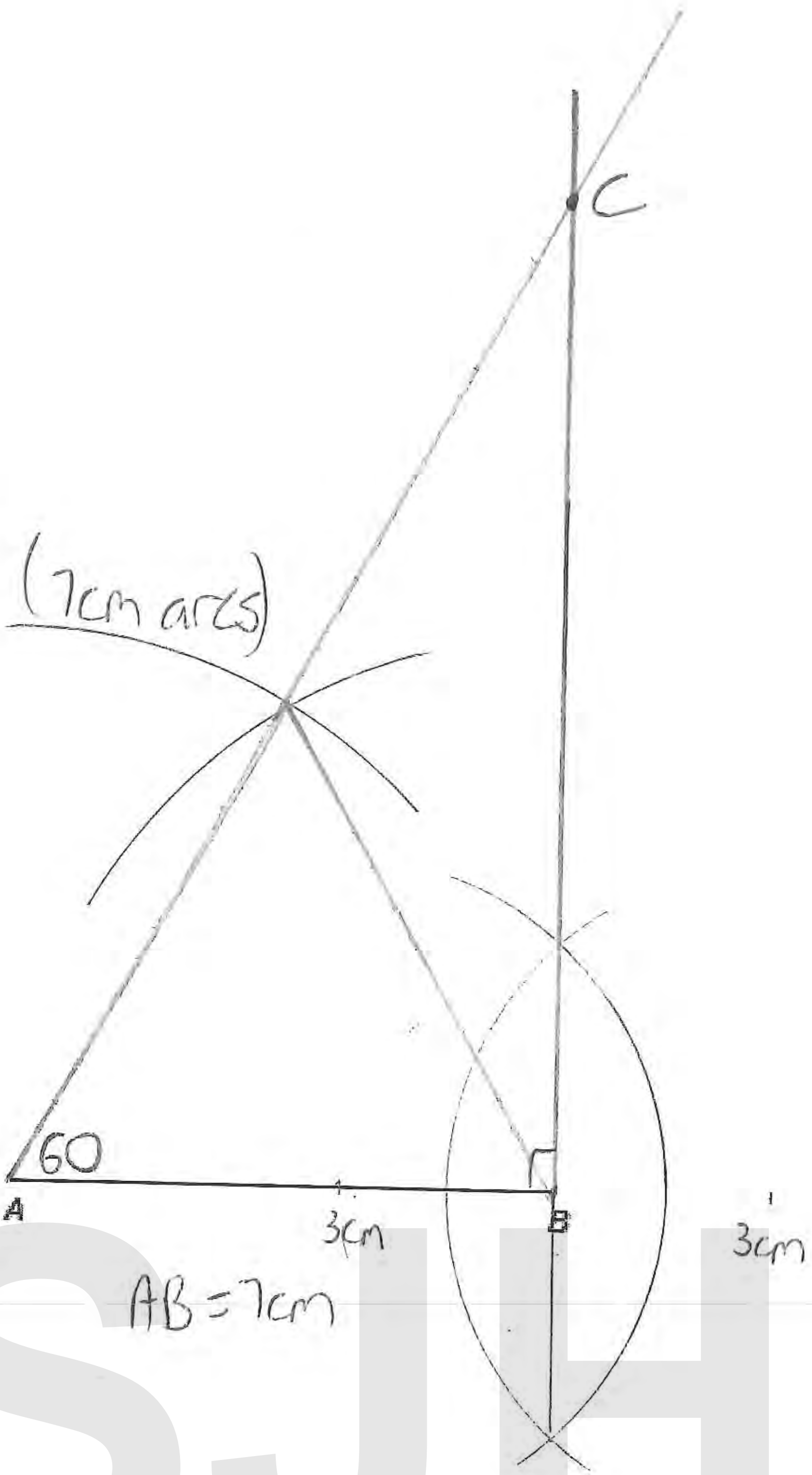


Using only a ruler and a pair of compasses, construct a perpendicular line from the point P to the line AB. [3]



Construct an accurate drawing of triangle ABC, where $AB = 7$ cm, $A\hat{B}C = 90°$ and $B\hat{A}C = 60°$.
Use only a ruler and a pair of compasses.
The side AB has been drawn for you.
You must show your construction arcs.

[3]



PQ and *PR* are tangents to a circle with centre *O*.
$R\hat{P}Q = 30°$.

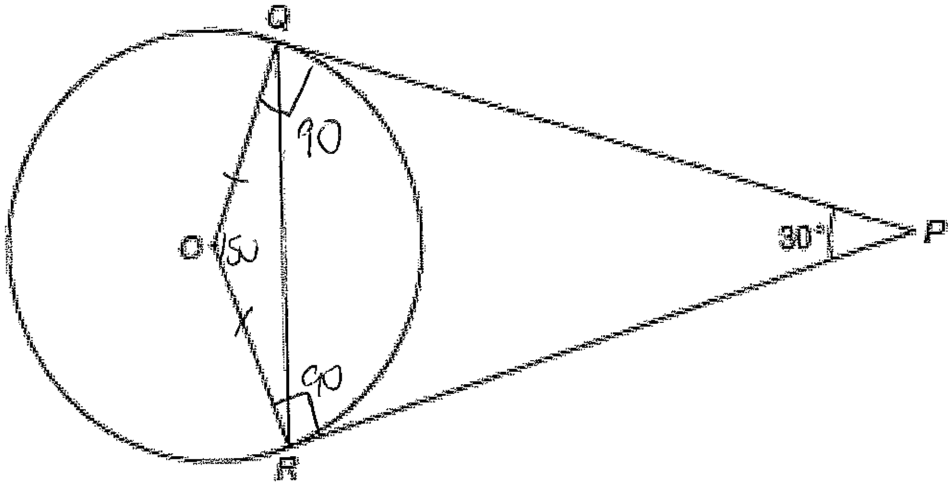


Diagram not drawn to scale

Find the size of $O\hat{Q}R$.

You must indicate any angles you calculate.
You must give a reason for each stage of your working. [5]

Tangent radius Angle between = 90°

QOR = 360 − 90 − 90 − 30   angles in a quad add to 360°

= 150°

QOR is isosceles as 2 radii

so OQR = $\frac{180-150}{2}$ = 15°

Points A, B, C and D lie on the circumference of a circle, centre O.
BD is a diameter of the circle.
The straight line $BC = 4{\cdot}7$ cm and $B\hat{A}C = 28°$.

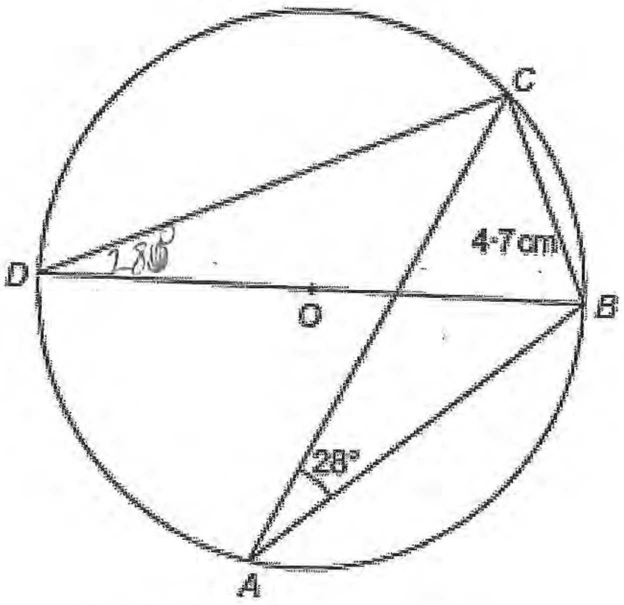
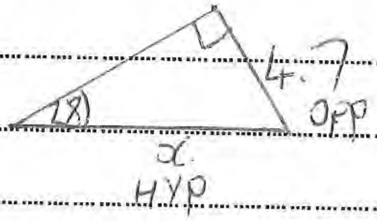


Diagram not drawn to scale

Write down the size of $B\hat{D}C$.
Hence, calculate the length BD.
You must show all your working. [5

$\hat{D} = 28°$ as $A = D$ as opposite same chord
$B\hat{C}D = 90°$ as opposite diameter



S O H

$$x = \frac{4.7}{\sin 28}$$

$x = 10$ cm

Points A, B and C lie on the circumference of a circle, centre O.

$A\hat{C}B = 37°$.

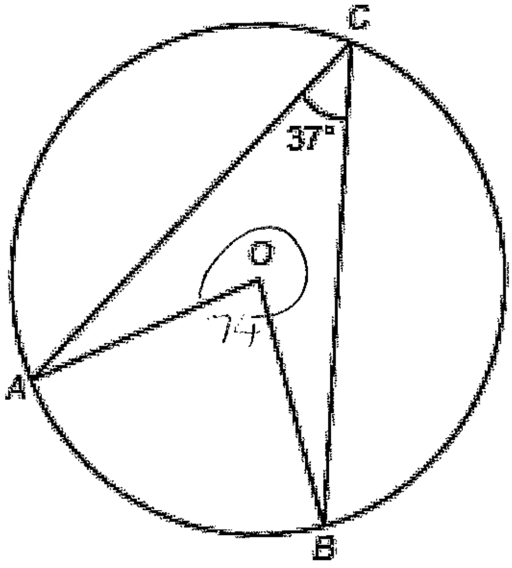


Diagram not drawn to scale

Calculate the size of the reflex angle $A\hat{O}B$. [2]

$360 - 74 = 286°$

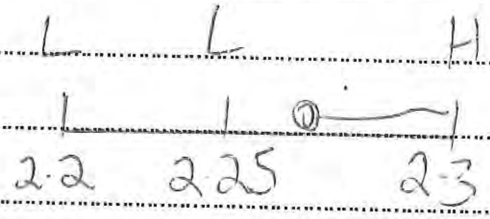
A solution to the equation

$$2x^3 - 3x - 17 = 0$$

lies between 2 and 3.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working. [4

x	$2x^3-3x-17$	H or L $=0$
2.5	6.75	H
2.4	3.448	H
2.3	0.434	H
2.2	-2.304	L
proof 2.25	-0.96875	L



$x = 2.3$

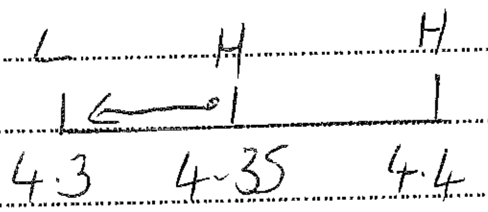
A solution of the equation

$$x^3 + 2x = 91$$

lies between 4 and 5.

Use the method of trial and improvement to find this solution correct to 1 decimal place. [4]
You must show all your working.

	x	$x^3 + 2x$	H or L 91
	4.5	100.125	H
	4.4	93.984	H
	4.3	88.107	L
proof	4.35	91.01	H



$\cancel{x < 91.}$

$x = 4.3$

. A solution to the equation

$$x^3 - 2x - 45 = 0$$

lies between 3 and 4.

Use the method of trial and improvement to find this solution correct to 1 decimal place.
You must show all your working. [4]

x	$x^3 - 2x - 45$	H or L = 0
3.5	-9.125	L
3.6	-5.544	L
3.7	-1.747	L
3.8	2.272	H
3.75	0.234375	H

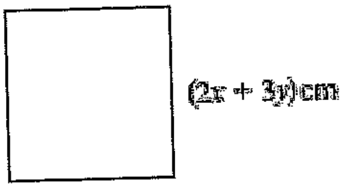
Proof

L H H

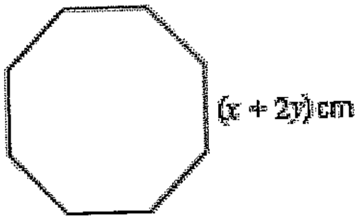
3.7 3.75 3.8

$x = 3.7$

Each side of a square is of length $(2x + 3y)$ cm.
The perimeter of the square is 62 cm.



Each side of a regular octagon is of length $(x + 2y)$ cm.
The perimeter of the octagon is 72 cm.



Use an algebraic method to find the value of x and the value of y. [5]

$4(2x+3y) = 62$ $\qquad$ $8(x+2y) = 72$

$8x + 12y = 62$ ① $\qquad$ $8x + 16y = 72$ ②

② − ① $\quad 4y = 10$

$y = 2.5$ cm $\quad$ sub in ①

$8x + 12 \times 2.5 = 62$

$8x + 30 = 62$

$8x = 32$

$x = 4$

$x =$ 4 $\qquad$ $y =$ 2.5

Solve the following simultaneous equations using an algebraic (not graphical) method. [4]

$$3x + 4y = 7 \quad \times 3 \quad (1)$$
$$2x - 3y = 16 \quad \times 4 \quad (2)$$

DSA

$9x + 12y = 21$ (3)

$8x - 12y = 64$ (4)

(3) + (4) $17x = 85$

$x = 5$ sub in (1)

$3 \times 5 + 4y = 7$

$15 + 4y = 7$

$4y = -8$

$y = -2$

Solve the following simultaneous equations using an algebraic (not graphical) method. [4]

$$4x - 3y = 2 \quad ① \times 5$$
$$6x - 5y = 1 \quad ② \times 3$$

SSS

$20x - 15y = 10$ ③
$18x - 15y = 3$ ④

③ - ④ $2x = 7$
$x = 3.5$ sub in ①

$4 \times 3.5 - 3y = 2$
$14 - 3y = 2$
$-3y = -12$
$y = 4$

Calculate the length of the side QR in the triangle PQR shown below. [3]

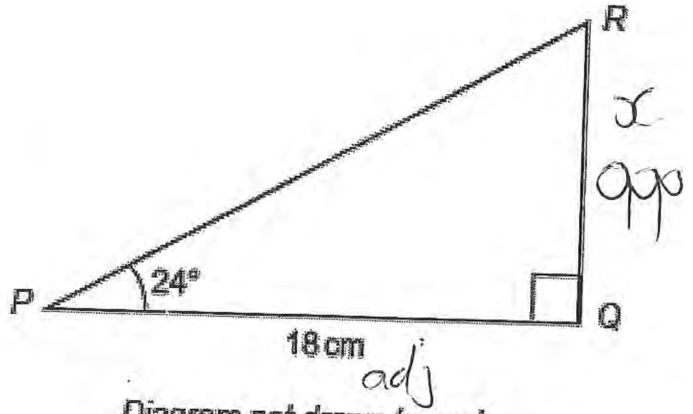


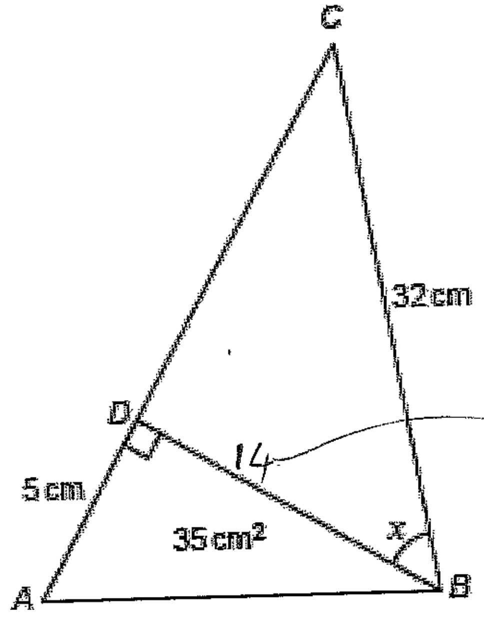
Diagram not drawn to scale

T O A

$x = \tan 24 \times 18$

$x = 8.01\text{cm}$

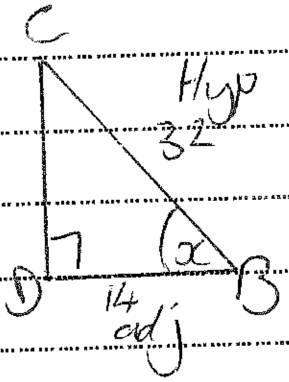
The area of triangle ABD, shown in the diagram below, is 35 cm^2.
$AD = 5\text{ cm}$ and $BC = 32\text{ cm}$.
D is on the line AC, and BD is perpendicular to AC.



$$\frac{14 \times 5}{2} = 35$$

Diagram not drawn to scale

Calculate the size of angle x.
You must show all your working. [5]



$$\cos x = \frac{14}{32}$$

$\cos^{-1}$

$x = 64.1°$

The diagram shows two right-angled triangles, joined together along a common side.
$S\hat{P}Q = 90°$, $S\hat{Q}R = 90°$, $S\hat{Q}P = 38°$, $PS = 8$ cm and $QR = 15$ cm.

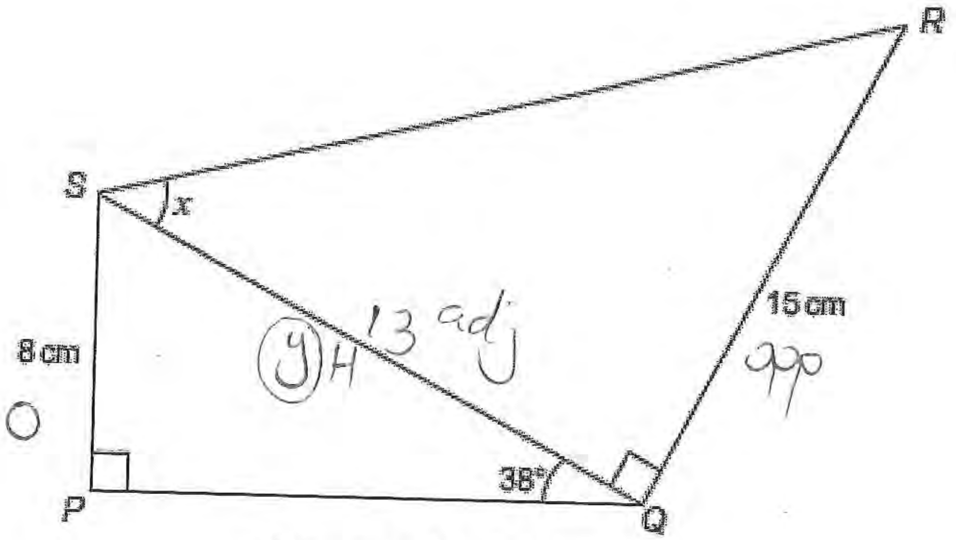


Diagram not drawn to scale

Calculate the size of angle x. [6]

O
S H

$y = \dfrac{8}{\sin 38}$

$y = 13$

O
T A

$\tan x = \dfrac{15}{13}$

$\tan^{-1}$

$x = 49.1°$

12. Show that the triangle below is **not** a right-angled triangle. [5]

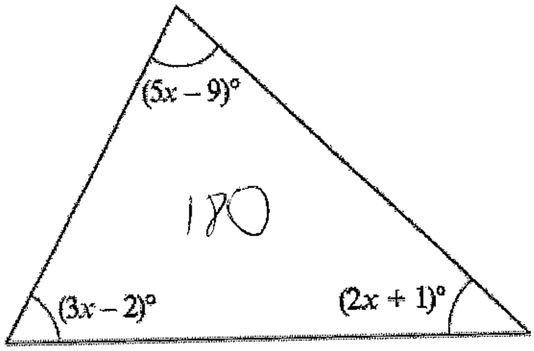


Diagram not drawn to scale

$5x - 9 + 3x - 2 + 2x + 1 = 180$

$10x - 10 = 180$

$10x = 190$

$x = 19$

$5x - 9 = 86$

$2x + 1 = 39$ None of those $= 90°$

$3x - 2 = 55$ so not right angled

6. *(a)* Write down the first three terms of the sequence whose nth term is given by $2n - 5$. [2]

$2 \times 1 - 5 = -3$

$2 \times 2 - 5 = -1$

$2 \times 3 - 5 = 1$

The first three terms are -3, -1 and 1

(b) Write down an expression for the nth term of the following sequence. [2]

3 7, 11, 15, 19, ...

$4n + 3$

18. *(a)* Factorise $x^3 - 5x$. [1]

$x(x^2-5)$

(b) Expand and simplify $(2x-3)(x+4)$. [2]

$\cancel{2x^2+8}$ $2x^2+5x-12$

(c) Factorise $x^2 - 3x - 28$. [2]

$(x-7)(x+4)$

10. *(a)* Write down the nth term of the following sequence. [2]

3, 4, 5, 6, ...

$n+2$

(b) The nth term of a different sequence is given by $n^2 + 7$.

(i) Write down the first three terms of this sequence. [2]

1^2+7

2^2+7

3^2+7

1st term = 8 2nd term = 11 3rd term = 16

(ii) Which term in this sequence is the first that has a value greater than 85? [2]

$9^2+7>85$

$81+7>85$ ✓

9th

12. Circle the correct answer for each of the following.

(a) $x^3 \times x^6 =$ [1]

x^{36} $x^{0.5}$ x^2 x^9 x^{18}

(b) $(7x - 5y) - (3x + 2y) =$ [1]

$7x - 5y - 3x - 2y$

$4x - 3y$ $4x - 7y$ $4x + 3y$ $-4x + 7y$ $-4x - 7y$

$4x - 7y$

18. (a) Factorise $x^2 - 2x - 24$, and hence solve $x^2 - 2x - 24 = 0$. [3]

$(x-6)(x+4)=0$

$x=6$ or $x=-4$

(b) Solve the equation $\frac{4x-3}{2} + \frac{7x+1}{6} = \frac{29}{2}$. [4]

$\times 3$ $\times 3$

$\frac{12x-9}{6} + \frac{7x+1}{6} = \frac{87}{6}$

$12x - 9 + 7x + 1 = 87$

$19x - 8 = 87$

$19x = 95$

$x = 5$

9. *ABC* is an isosceles triangle with *AB* = *AC*.

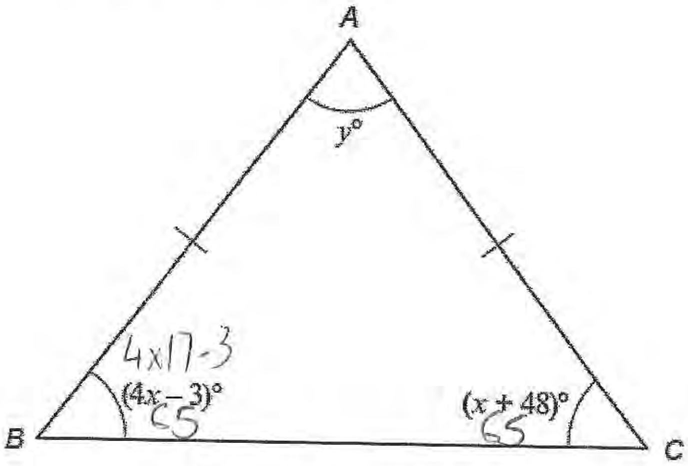


Diagram not drawn to scale

Calculate the value of y. [6]

$4x - 3 = x + 48$ $\quad$ $180 - 65 - 65$

$3x = 51$

$x = 17$ $\quad$ $y = 50°$

10. Simplify each of the following and circle the correct answer in each case.

(a) $6p^6 \times 3p^3$ [1]

$9p^9$ $\quad$ $9p^{18}$ $\quad$ $18p^{18}$ $\quad$ $18p^2$ $\quad$ $18p^9$

(b) $3{\cdot}4g^8 \div 13{\cdot}6g^2$ [1]

$\frac{g^4}{4}$ $\quad$ $\frac{g^6}{4}$ $\quad$ $4g^4$ $\quad$ $4g^6$ $\quad$ $0{\cdot}4g^6$

(c) $\frac{m^3 \times m^6}{m^9}$ $\quad$ $\frac{m^9}{m^9}$ [1]

1 $\quad$ m $\quad$ m^2 $\quad$ m^4 $\quad$ 4

14. (a) Rearrange the following formula to make x the subject. Give your answer in its simplest form. [3]

$$2(x + y) = 7y - 3$$

$2x + 2y = 7y - 3$

$2x = 5y - 3$

$x = \frac{5y-3}{2}$

(b) Write down the nth term of the following sequence. [2]

3, 6, 11, 18, 27, ...

1 4 9 16

$n^2 + 2$

(c) Solve $9x + 3 = 4x + 5$. [3]

$5x = 2$

$x = \frac{2}{5}$

x	−2	−1	0	1	2	3
$y = 2x^2 - 5$	3		−5	−3	3	13

15. Factorise $x^2 - 7x - 18$, and hence solve $x^2 - 7x - 18 = 0$. [3]

$(x-9)(x+2) = 0$

$x = 9$ or $x = -2$

3. A shop has 31 plant pots.
Some are blue, some are yellow and the rest are red.
There are five more blue pots than yellow pots.
There are four times as many blue pots as there are red pots.

Calculate how many pots there are of each colour. [3]

B Y R

15 10 x Blue is 4x table

16 11 4 = 31

Blue 16 Yellow 11 Red 4

x	−2	−1	0	1	2	3	4
$y = 2x^2 - 5x - 1$	17		−1	−4		2	11

[1]

(b) $\frac{42}{x} = 7$

6

[3]

(c) $13y - 5 = 9y + 27$

$4y = 32$

$y = 8$

x	−1	0	1	2	3	4	5
$y = x^2 - 5x + 2$	8	2	−2	−4		−2	2

[2]

13. (a) Make m the subject of the formula $y = 6m + 7$.

$y - 7 = 6m$

$\frac{y-7}{6} = m$

[2]

(b) Factorise $6x^2 - 12x$.

$6x(x-2)$

SJHS

17. William has n marbles.
Lois had 4 times as many marbles as William, but she has now lost 23 of them.

Lois still has more marbles than William.

Write down an inequality in terms of n to show the above information.
Use your inequality to find the least number of marbles that William may have. [4]

W L

n $4n-23$

$4n-23>n$

$3n>23$

$n>7\frac{2}{3}$

$n=8$ marbles

15. In the following formulae, each measurement of length is represented by a letter.

Consider the dimensions implied by the formulae.
Write down, for each case, whether the formula could be for a length, an area, a volume or none of these.

The first one has been done for you. [3]

Formula	Formula could be for
$d^3 - 3{\cdot}14r^2h$	volume
$d^2 + hw$	Area
$d + w + h$	length
$2\pi r - \pi r^2$	None
$(d + h)w$ dw + hw	Area
$d^3 + dwh$	Volume

B3

19. Rashid owned n sheep.
Eifion had exactly 4 times as many sheep as Rashid.

Rashid buys 17 extra sheep.
Eifion sells 8 of his sheep.

Eifion still has more sheep than Rashid.

Form an inequality, in terms of n.
Solve the inequality to find the least value of n. **[5]**
You must show all your working.

R E

n $4n$

$n+17$ $4n-8$

$4n-8 > n+17$

$3n > 25$

$n > 8\frac{1}{3}$

$n = 9$

Find, in standard form, the value of each of the following.

(a) $\frac{7 \cdot 5 \times 10^6}{5000}$ [2]

$\frac{7.5 \times 10^6}{5 \times 10^3} = 1.5 \times 10^3$

(b) $(2 \cdot 3 \times 10^3) + (6 \cdot 4 \times 10^4)$ [2]

$$\begin{array}{r} 64000 \\ +\ \ 2300 \\ \hline 66300 \end{array}$$

6.63×10^4

Calculate the value of $(5 \cdot 41 \times 10^5) + (2 \cdot 3 \times 10^4)$.
Give your answer in standard form. [2]

$$\begin{array}{r} 541000 \\ +\ \ 23000 \\ \hline 564000 \end{array}$$

5.64×10^5

(a) Express 0·00042 in standard form. [1]

4.2×10^{-4}

(b) Calculate the value of $\frac{7 \cdot 2 \times 10^6}{2 \times 10^{-2}}$.

Give your answer in standard form. [1]

3.6×10^8

(c) Calculate the value of $(4 \cdot 7 \times 10^5) - (6 \cdot 2 \times 10^4)$.
Give your answer in standard form. [2]

~~47~~

$$\begin{array}{r} 470000 \\ -\ \ 62000 \\ \hline 408000 \end{array}$$

$= 4.08 \times 10^5$

11. (a) The table below shows some of the values of $y = 2x^2 - 5x - 1$ for values of x from -2 to 4.

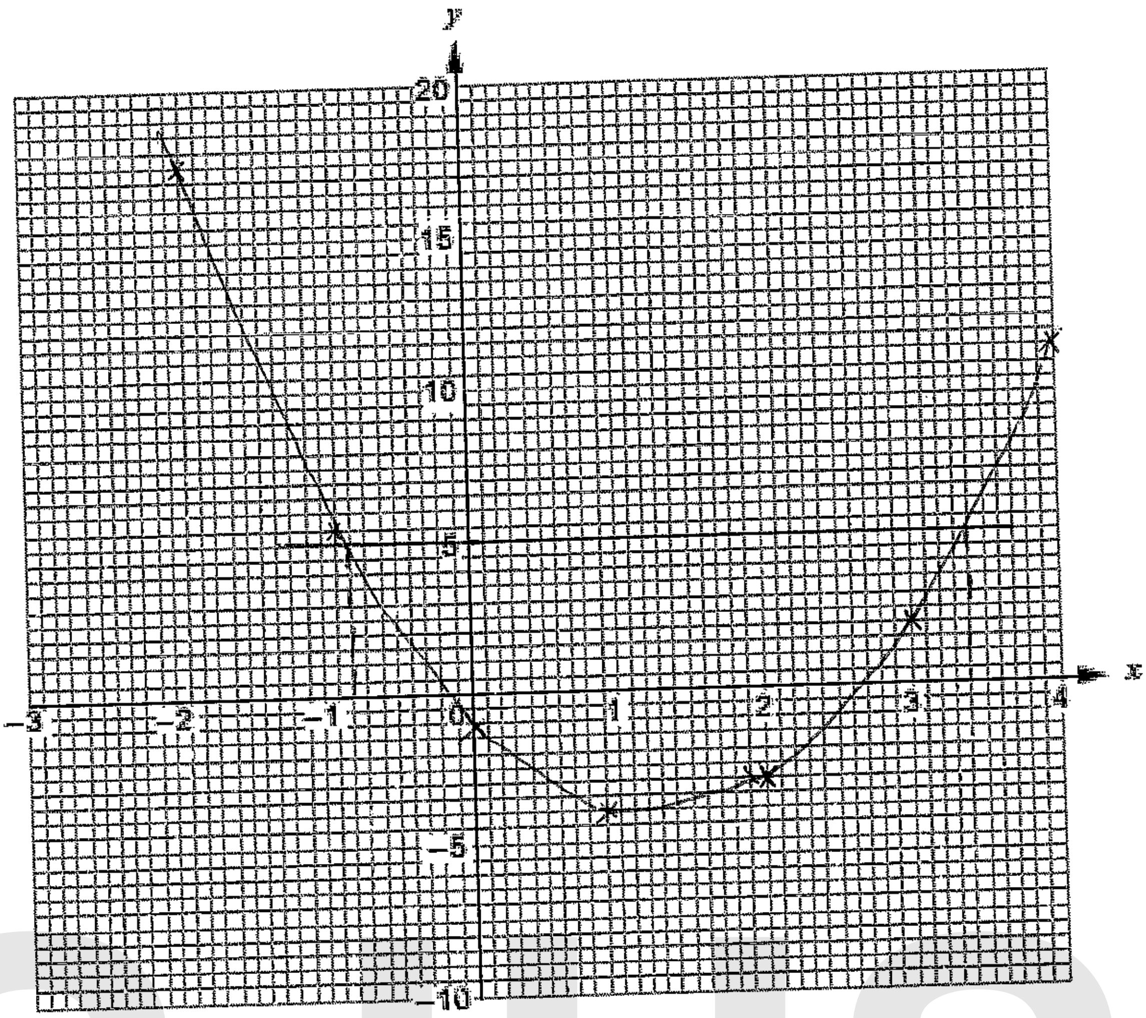
Complete the table by finding the value of y for $x = -1$ and for $x = 2$. [2]

x	-2	-1	0	1	2	3	4
$y = 2x^2 - 5x - 1$	17	6	-1	-4	-3	2	11

$2 \times 1 - 5 \times -1 - 1$ $\qquad$ $2 \times 4 - 5 \times 2 - 1$

$2 + 5 - 1$ $\qquad$ $8 - 10 - 1$

(b) On the graph paper below, draw the graph of $y = 2x^2 - 5x - 1$ for values of x from -2 to 4. [2]



PTO

(c) Draw the line $y = 5$ on the graph paper.

Write down the values of x where the line $y = 5$ cuts the curve $y = 2x^2 - 5x - 1$.
Give your answers correct to 1 decimal place. [2]

Values of x are -0.9 and 3.4

(d) Circle the equation below whose solutions are the values you have given in (c). [1]

$2x^2 - 5x - 1 = 0$ $\quad$ $2x^2 - 5x - 6 = 0$ $\quad$ $2x^2 - 5x - 5 = 0$

$2x^2 - x - 1 = 0$ $\quad$ $2x^2 - 5x + 4 = 0$

$2x^2 - 5x - 1 = 5$

$-5 \quad -5$

$2x^2 - 5x - 6 = 0$

SJHS

11. The table below shows some of the values of $y = x^2 - 5x + 2$, for values of x from -1 to 5.

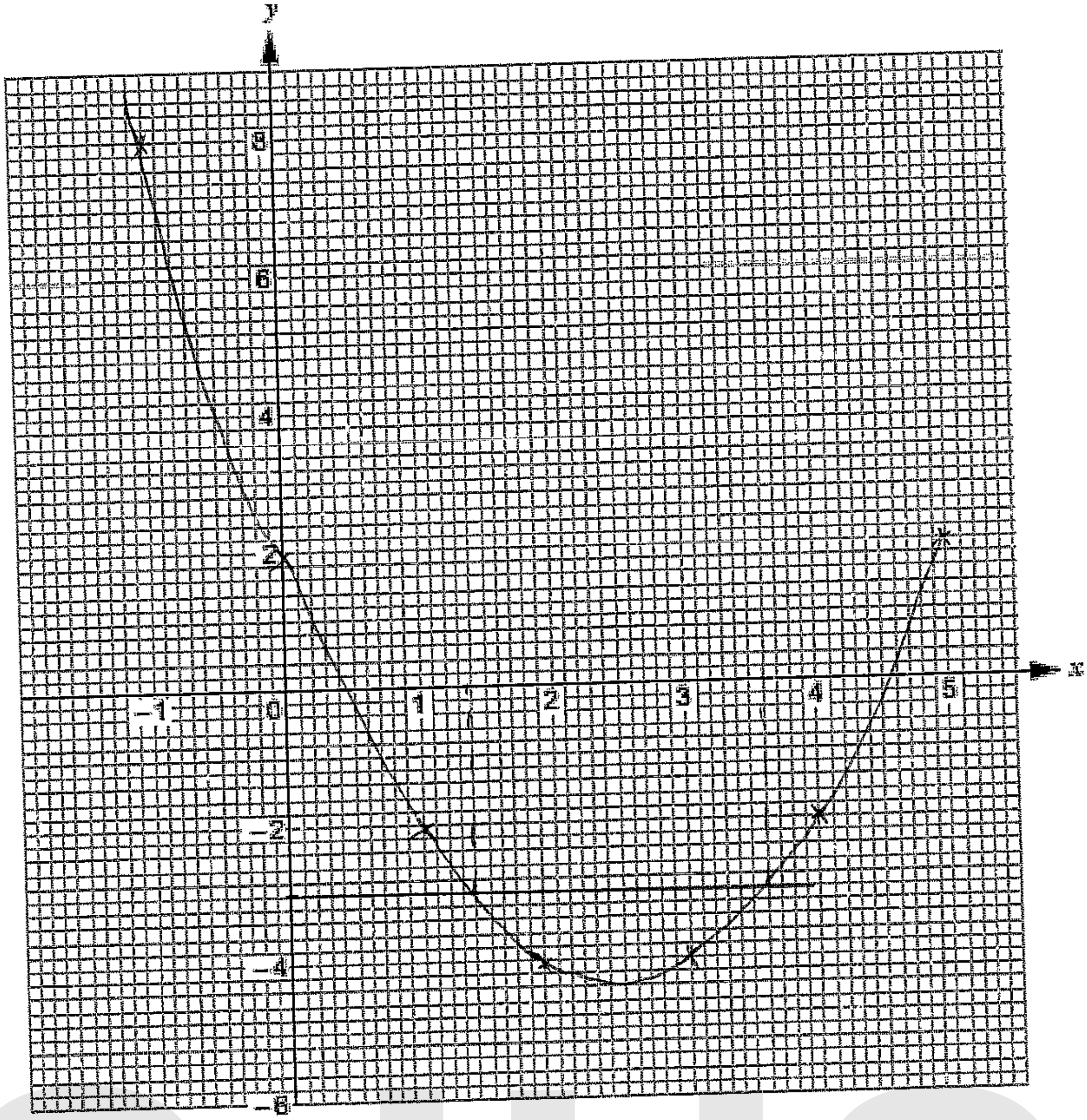
x	-1	0	1	2	3	4	5
$y = x^2 - 5x + 2$	8	2	-2	-4	-4	-2	2

(a) Complete the table above. [1]

$3^2 - 5 \times 3 + 2$

$9 - 15 + 2$

(b) On the graph paper below, draw the graph of $y = x^2 - 5x + 2$ for values of x from -1 to 5. [2]



PTO

(c) Draw the line $y = -3$ on the graph paper.

Write down the values of x where the line $y = -3$ cuts the curve $y = x^2 - 5x + 2$.
Give your answers correct to 1 decimal place. [2]

Values of x are 1.4 and 3.6

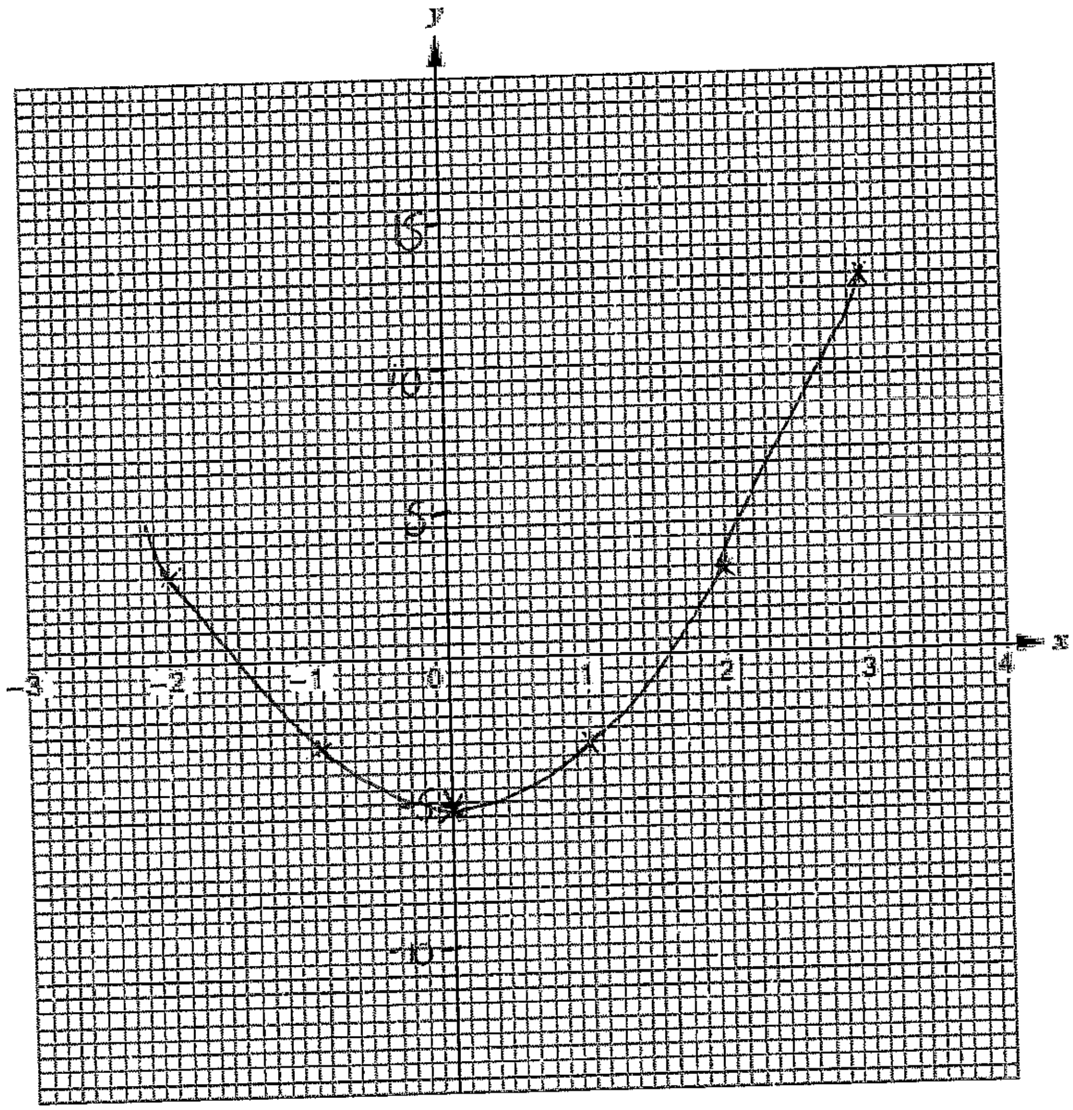
(a) Complete the table below.
Draw the graph of $y = 2x^2 - 5$ for values of x between -2 and 3.
Use the graph paper below.
Choose a suitable scale for the y-axis. [4]

x	-2	-1	0	1	2	3
$y = 2x^2 - 5$	3	-3	-5	-3	3	13

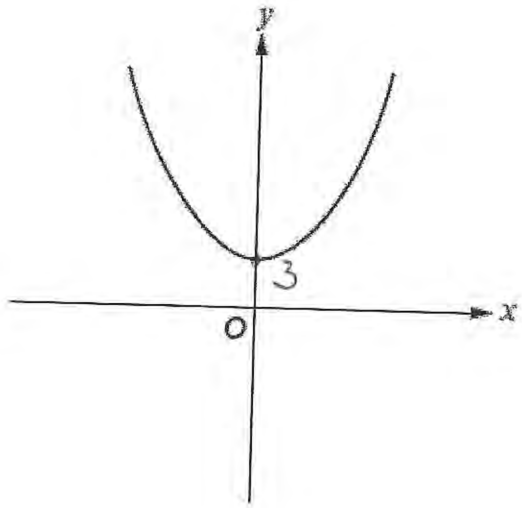
$2 \times (-1)^2 - 5$

$2 \times 1 \quad - 5$

$2 - 5 = -3$



(b)



The sketch above can represent only one of the equations given below.
Circle this equation. [1]

$y = x^2$ $\quad$ $y = x^2 - 3$ $\quad$ $y = -x^2$ $\quad$ $y = x^2 + 3$ $\quad$ $y = 3x$

10. The radius of a hemisphere and the radius of a cylinder are equal.
The hemisphere and cylinder have equal volumes.

Calculate the ratio of the height of the cylinder to the radius of the cylinder. [3]

H C

$\frac{2}{3}\pi r^3 = \pi r^2 h \quad \div \pi$

$\frac{2}{3}r^3 = r^2 h \quad \div r^2$

$\frac{2}{3}r = h$

$2r = 3h$

height of cylinder : radius of cylinder

= 1 : $\frac{3}{2}$

19. By considering algebraic expressions, show that it will never be possible for the surface area of a sphere of radius r to be equal to the surface area of a cube with sides of length r. [2]

$4\pi r^2 = 6r^2$

$4\pi = 6 \quad \div r^2$

$4\pi \neq 6$ so it is not possible

SJHS

10. A cylinder just fits inside a hollow cube with sides of length m cm.

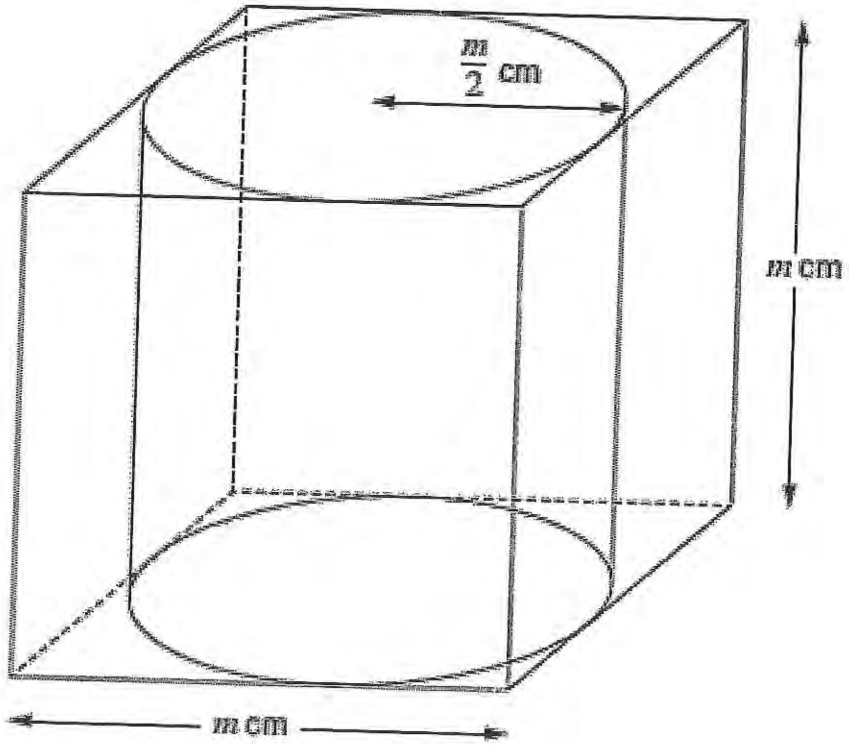


Diagram not drawn to scale

The radius of the cylinder is $\frac{m}{2}$ cm.

The height of the cylinder is m cm.

The ratio of the volume of the cube to the volume of the cylinder is given by

volume of cube : volume of cylinder

$= k : \pi,$

where k is a number.

Find the value of k.
You must show all your working. [4]

Cube Cylinder

$m^3 : \pi\left(\frac{m}{2}\right)^2 \times m$

$m^3 : \pi \frac{m^3}{4} \quad \div m^3$

$1 : \pi/4$

$4 : \pi$

$k = 4$

4. A triangular prism of length 2 metres is shown below.

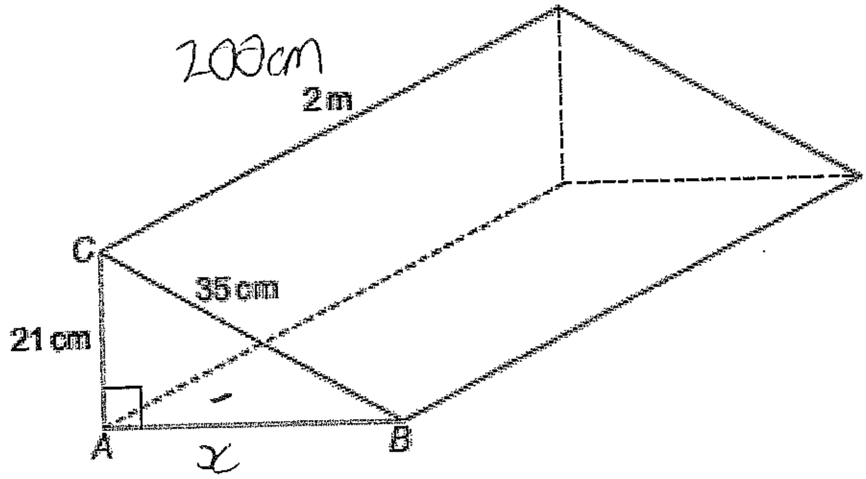


Diagram not drawn to scale

$AC = 21$ cm, $BC = 35$ cm and $B\hat{A}C = 90°$.

(a) *In this part of the question, you will be assessed on the quality of your organisation, communication and accuracy in writing.*

Calculate the area of triangle ABC.
Give your answer in cm^2.
You must show all your working. [5 + 2 OCW]

Area Triangle = $\frac{1}{2}$ b×h

Use Pythagoras to find base of triangle $x^2 = 35^2 - 21^2$ as we need a smaller side

$x^2 = 784$

$x = 28$

Area = $\frac{1}{2} \times 28 \times 21$

$= 294 cm^2$

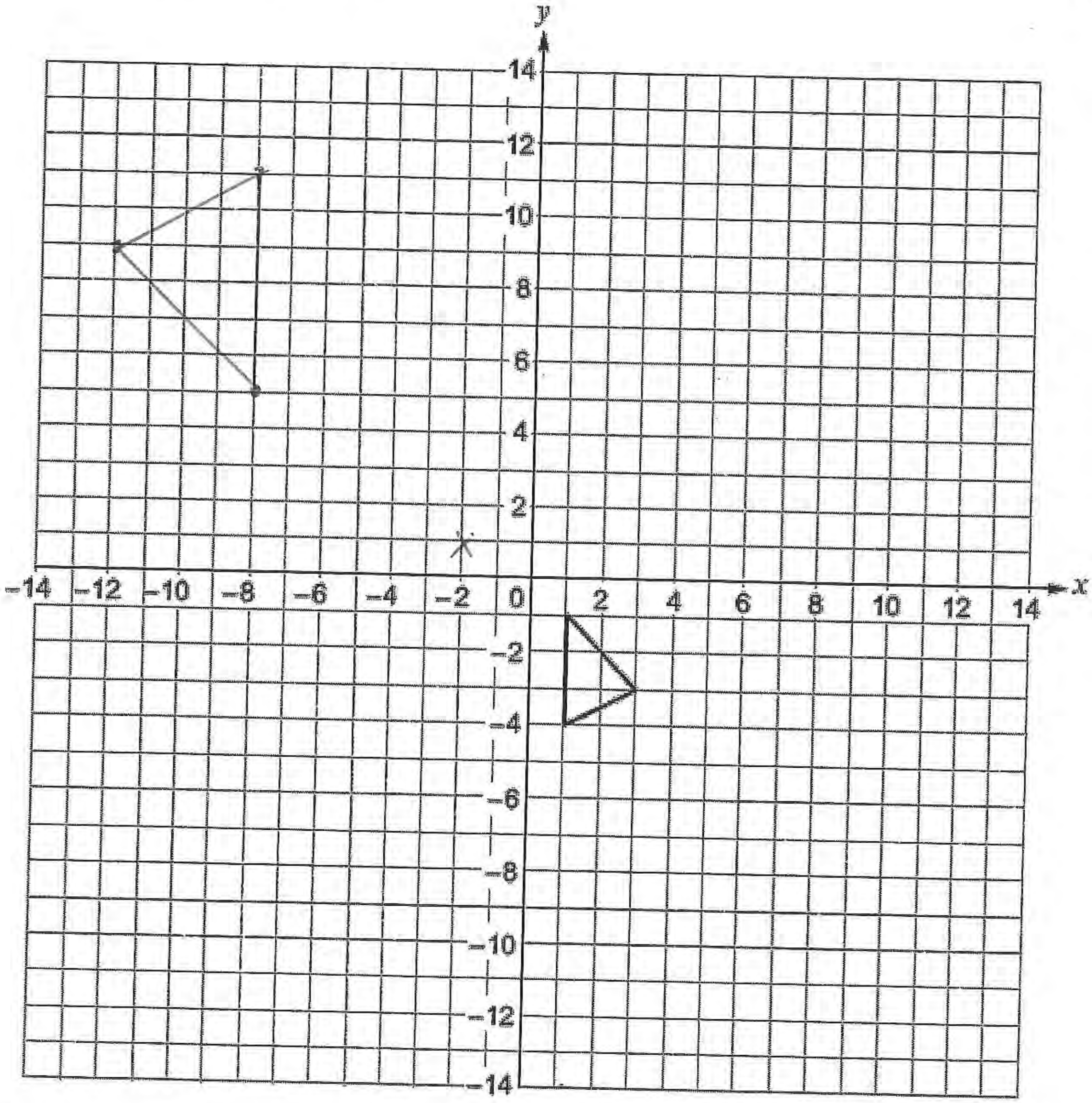
(b) Calculate the volume of the prism.
You must give the units of your answer. [3]

V = CSA × L

V = 294 × 200cm

V = 58800 cm^3

10. Draw the enlargement of the given triangle, using
- a scale factor of –2,
- (–2, 1) as the centre of enlargement. [3]



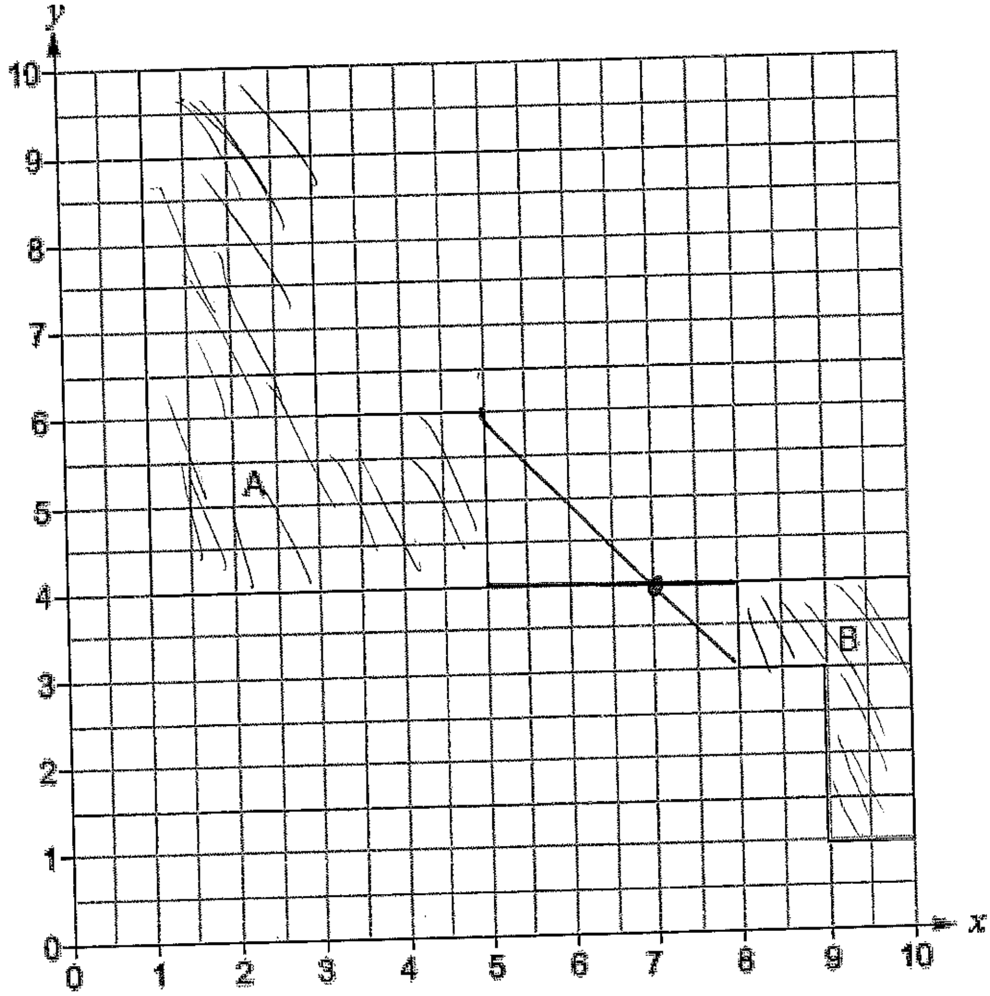
~~3← 2↑ = 6→ 4↓~~

3→ 2↓ = 6← 4↑

5→ 4↓ = 10← 8↑

3→ 5↓ = 6← 10↑

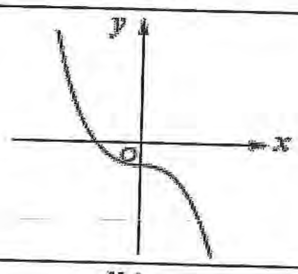
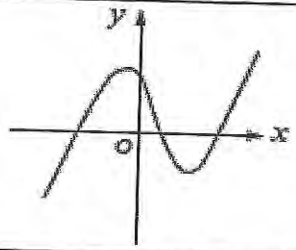
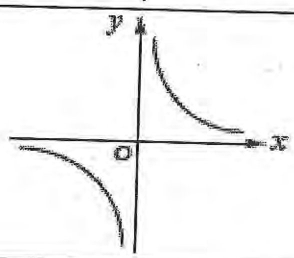
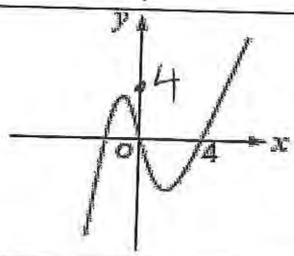
15. Describe fully a single transformation that transforms shape A onto shape B. [3]



Enlargement scale factor $-\frac{1}{2}$

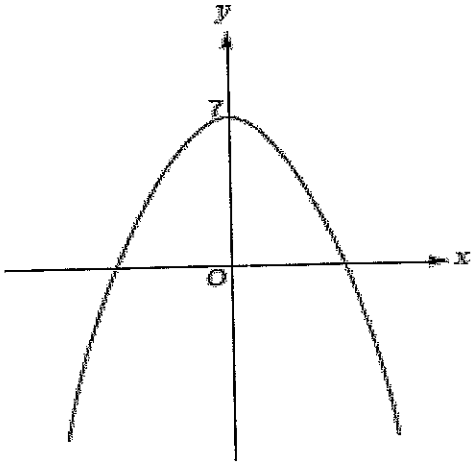
(Gets smaller and rotate) about (7,4)

15. Circle either TRUE or FALSE for each statement given below. [2]

GRAPH	STATEMENT		
	The equation of this graph could be $y = -x^3 - 2$.	TRUE	FALSE
	The equation of this graph could be $y = x^3 - 9x$. NOT THROUGH O	TRUE	FALSE
	The equation of this graph could be $y = x^{-1}$.	TRUE	FALSE
	The equation of this graph could be $y = x^3 + 4$.	TRUE	FALSE

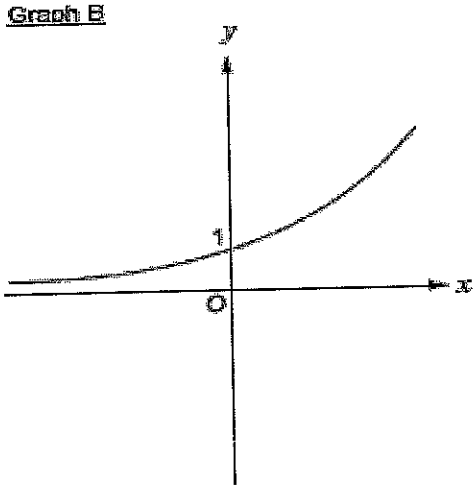
16. Each of the two graphs below is described by one of the equations on the right. Put a tick in the box next to the equation which correctly describes each graph. [2]

Graph A



	Equation describing graph A
$y = 7x^2$	
$y = -(x+7)^2$	
$y = (x-7)^2$	
$y = 7 - x^2$	✓
$y = x^2 + 7$	

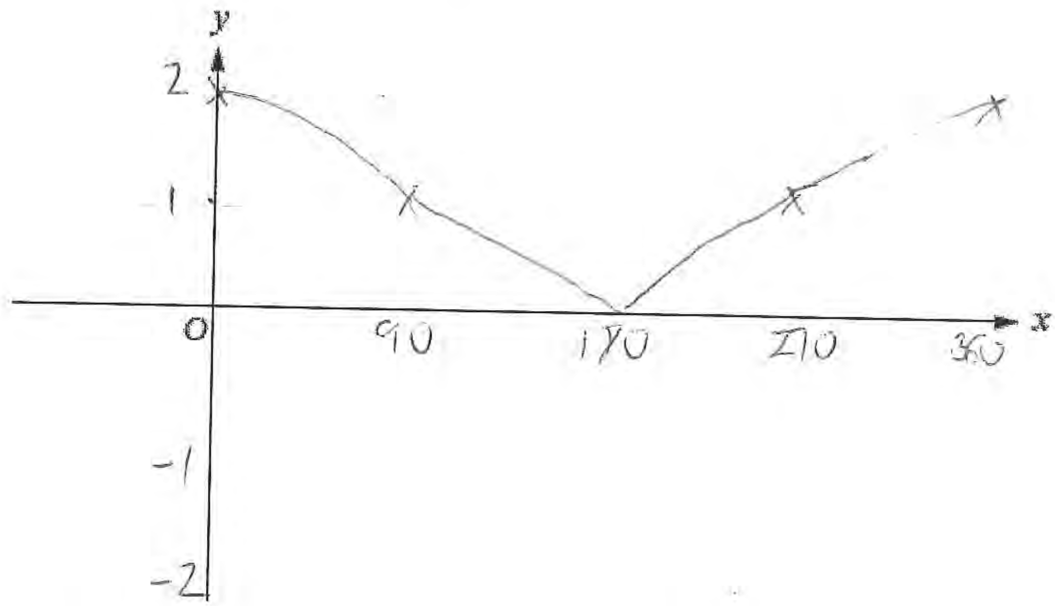
Graph B



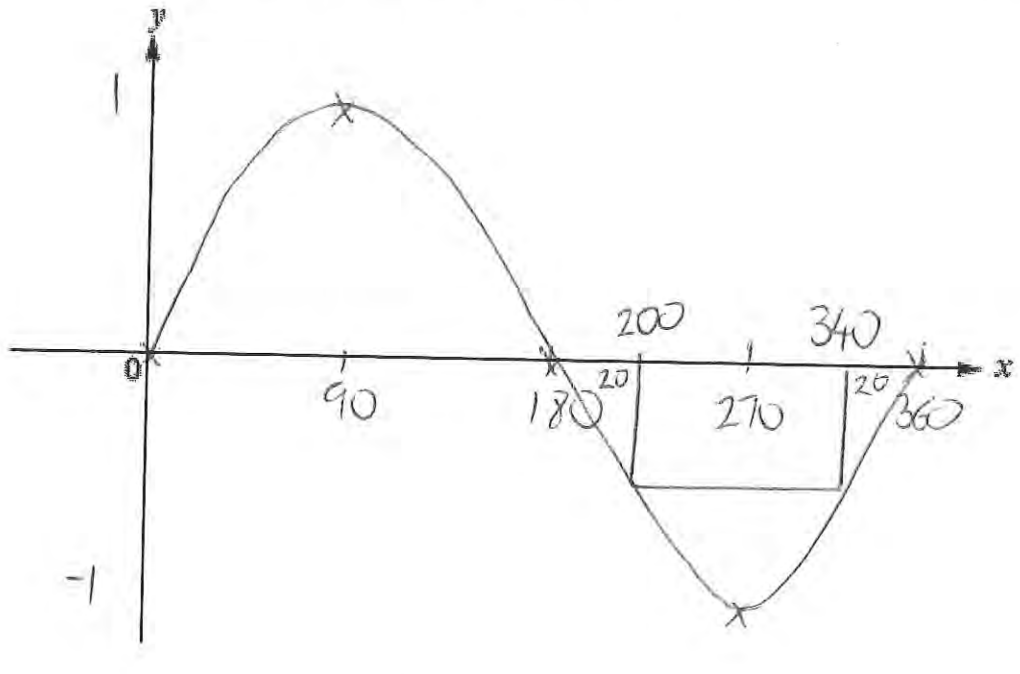
	Equation describing graph B
$y = x^2 + 1$	
$y = 2^x$	✓
$y + 1 = x^2$	
$y = \frac{1}{x}$	
$y = x^6$	

....

(b) Using the axes below, **sketch** the graph of $y = \cos x + 1$ for values of x from 0° to 360
[



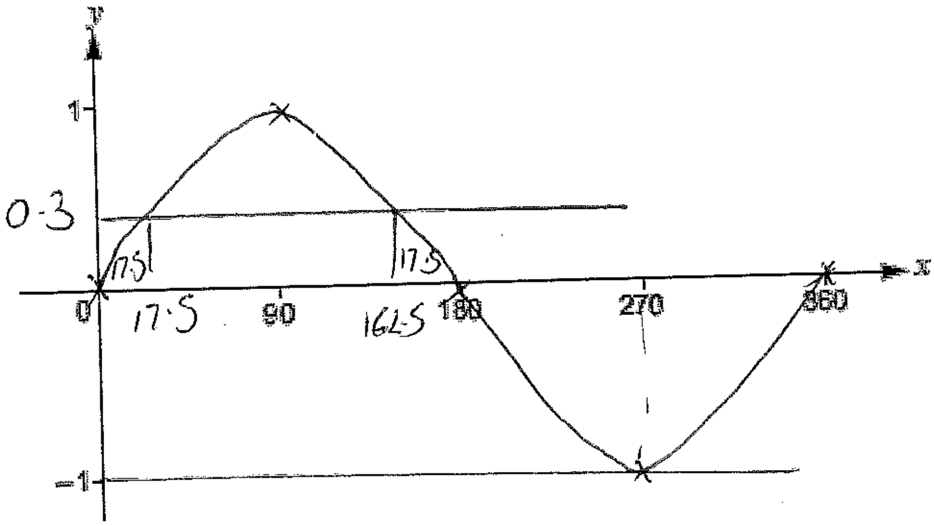
15. (a) Using the axes below, **sketch** the graph of $y = \sin x$ for values of x from 0° to 360°.
You must label any important values on both axes. [2]



(b) Circle the value that is equal to sin 200°. [1]

sin 20° sin 100° sin 160° sin 220° sin 340°

14. (a) Sketch the curve $y = \sin x$, for values of x in the range $x = 0°$ to $x = 360°$. [1]



(b) Solve each of the following equations.
Give all answers in the range $x = 0°$ to $x = 360°$.

(i) $\sin x = 0·3$ [2]

$\sin^{-1}$ 180-17·5

$x = 17·5°, 162·5°$

(ii) $\sin x + 1 = 0$ [1]

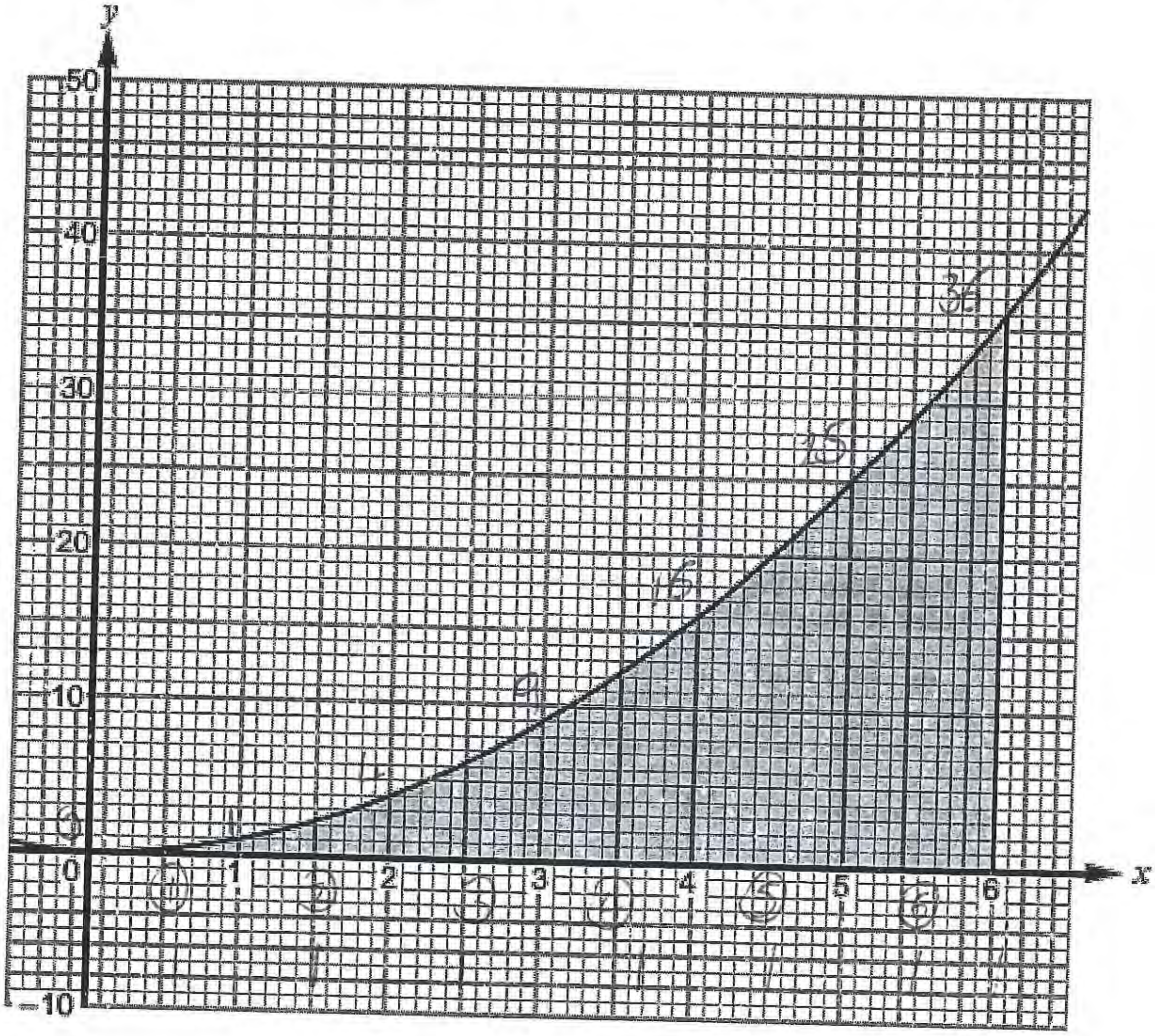
$\sin x = -1$

$\sin^{-1}$

$x = 270$

WJEC Higher Tier Trapezium rule ↓

18. The graph of $y = x^2$ has been drawn below, for values of x from $x = 0$ to $x = 6$.



Use the trapezium rule, with the ordinates $x = 0$, $x = 1$, $x = 2$, $x = 3$, $x = 4$, $x = 5$ and $x = 6$, to estimate the area of the shaded region shown above. [4

① $A = \left(\frac{0+1}{2}\right) \times 1 = 0.5$

② $A = \left(\frac{1+4}{2}\right) \times 1 = 2.5$

③ $A = \left(\frac{4+9}{2}\right) \times 1 = 6.5$

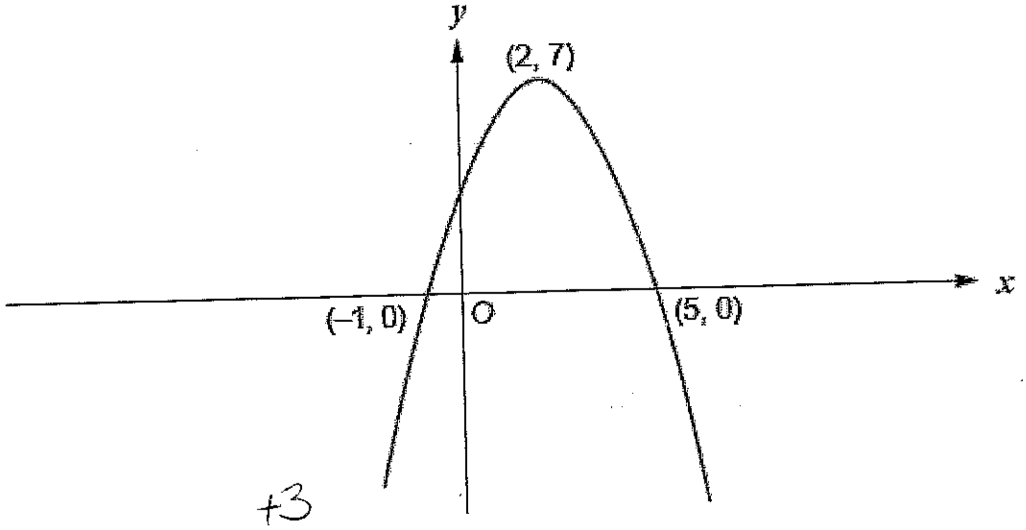
④ $\left(\frac{9+16}{2}\right) \times 1 = 12.5$

⑤ $\left(\frac{16+25}{2}\right) \times 1 = 20.5$

⑥ $\left(\frac{25+36}{2}\right) \times 1 = 30.5$

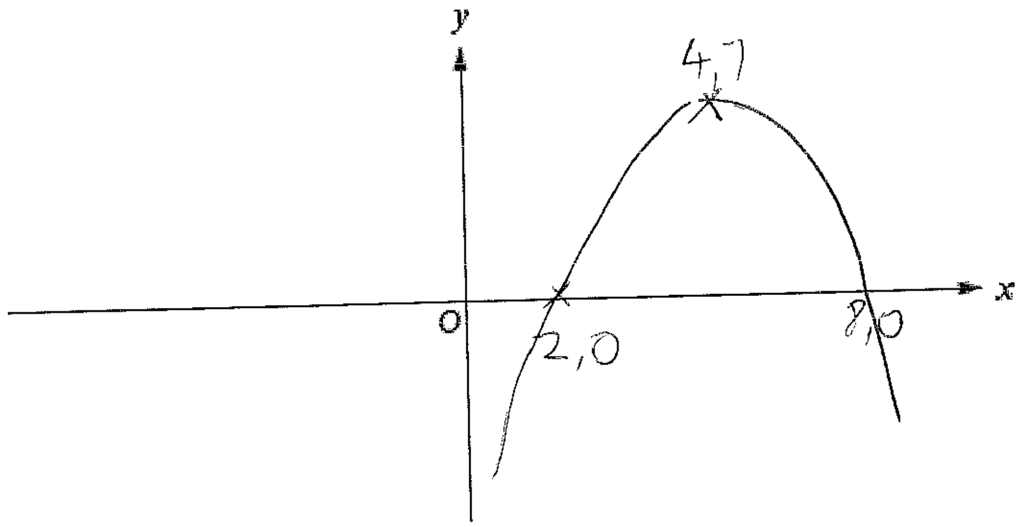
Total area $= 73$

15. *(a)* The diagram shows a sketch of the graph $y = f(x)$.
The graph passes through the points $(-1, 0)$ and $(5, 0)$ and its highest point is at $(2, 7)$.

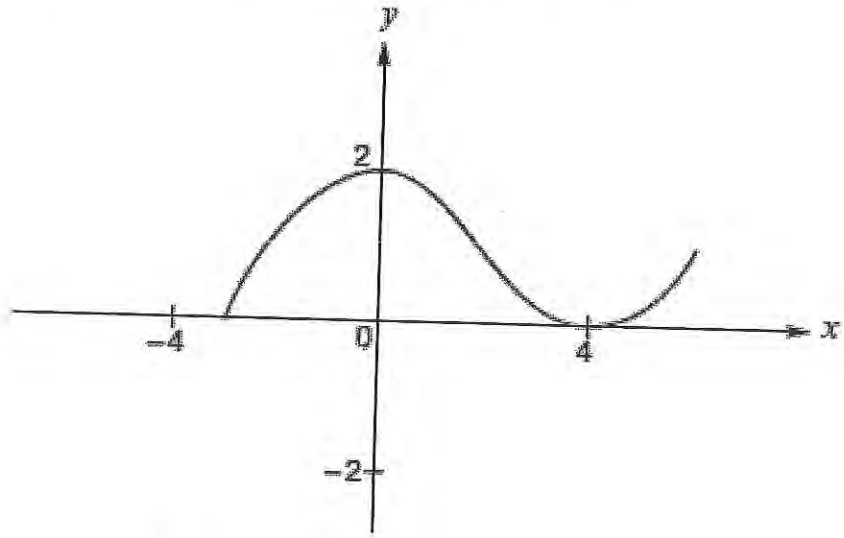


+3

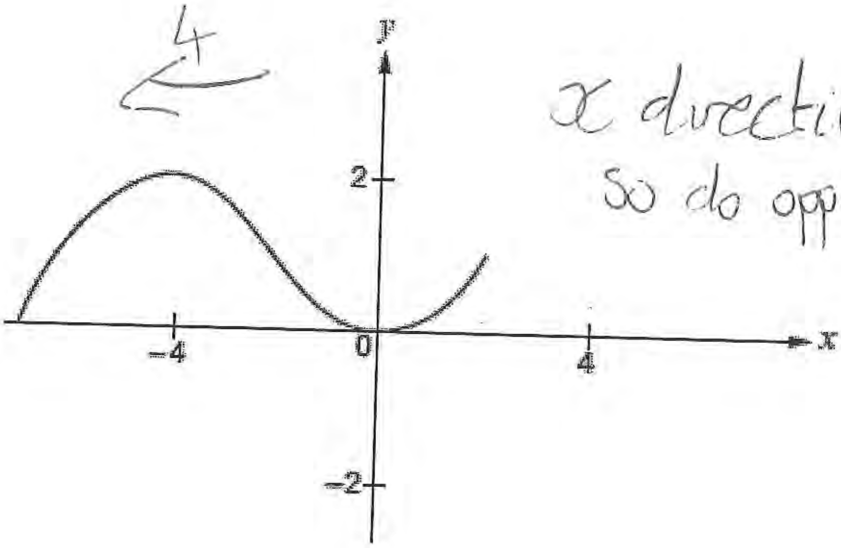
Sketch the graph of $y = f(x - 3)$ on the axes below.
You must indicate
- the coordinates of the points of intersection of the graph with the x-axis
- the coordinates of the highest or lowest point. [3]



18. The following diagram shows a sketch of the curve $y = f(x)$.



The curve is transformed, as shown below.



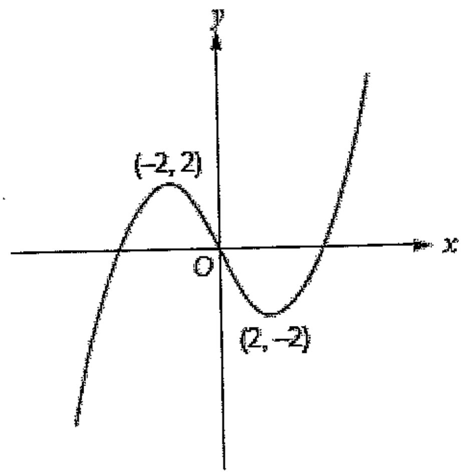
x direction
so do opposite

Using function notation, complete the following to give the equation of the transformed curve. [1]

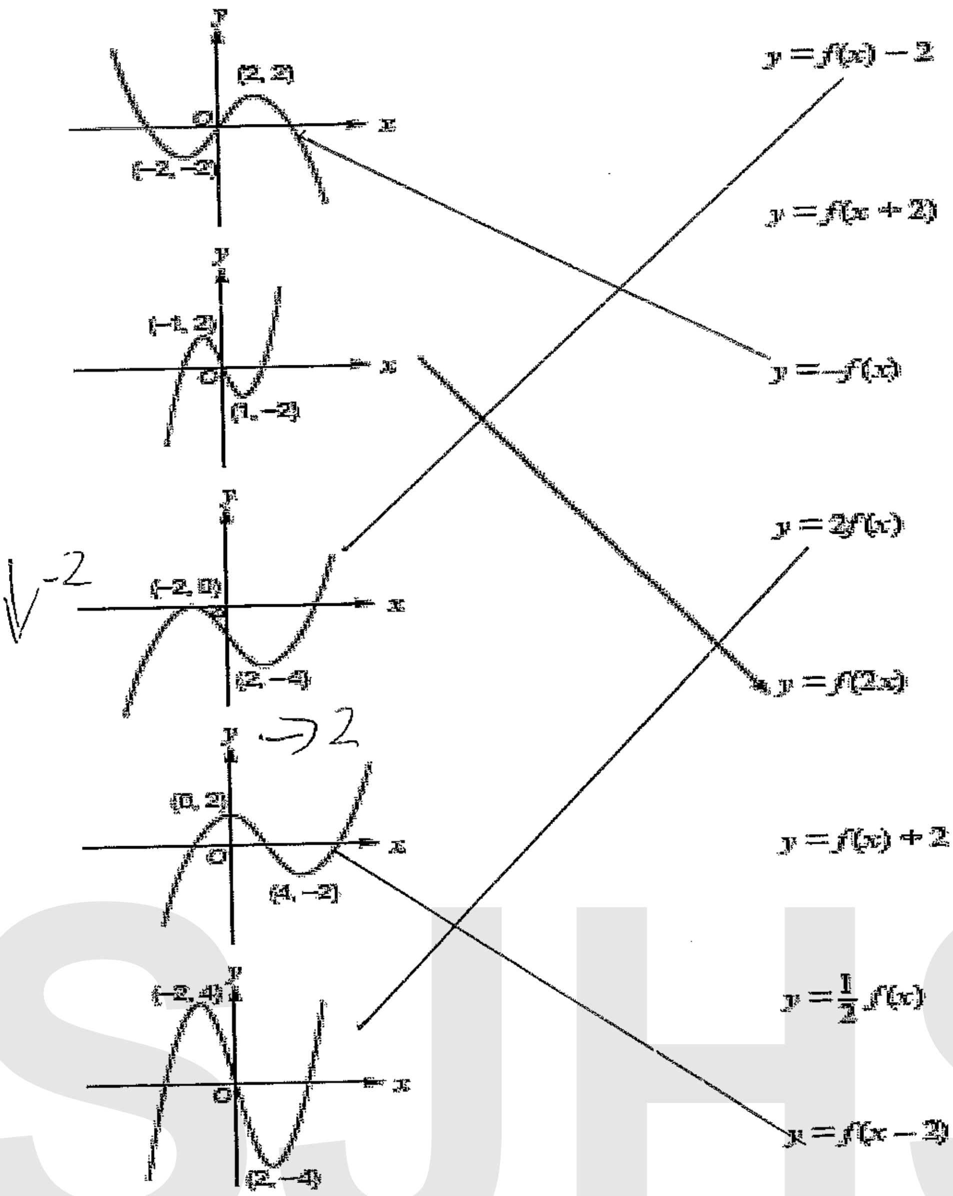
The equation of the transformed curve is

$y =$ $f(x+4)$

20. A sketch of the graph $y = f(x)$ is shown below.
Two specific points are shown on the graph. They are called a maximum point and a minimum point.
The maximum point shown is $(-2, 2)$ and the minimum point shown is $(2, -2)$.



The graphs on the opposite page are transformations of $y = f(x)$.
Draw a line connecting each graph to the equation describing the transformation.
One has been done for you. [4]



17. Simplify

$$\frac{\left(5\sqrt{3}\right)^2 - \frac{2\sqrt{18}}{\sqrt{2}}}{\sqrt{32} \times \sqrt{2}}$$

and state whether your answer is rational or irrational. [5]

$\dfrac{25 \times 3 - 2\sqrt{9}}{\sqrt{64}}$ $\quad \dfrac{75 - 6}{8} = \dfrac{69}{8}$

RATIONAL

(c) Find the value of $\left(\sqrt{63} - \sqrt{7}\right)^2$. [3]

$\times$	$\sqrt{63}$	$-\sqrt{7}$
$\sqrt{63}$	63	$-\sqrt{441}$
$-\sqrt{7}$	$-\sqrt{441}$	7

$\sqrt{441} = 21$

$63 - 21 - 21 + 7 = 48$

16. You are given that $p = \sqrt{40}$ and $q = \sqrt{10}$.
Circle the correct answer in each of the following:

[1]

(a) p is equal to

$10\sqrt{4}$ $\quad$ $4\sqrt{10}$ $\quad$ $10\sqrt{2}$ $\quad$ $2\sqrt{10}$ $\quad$ 20

$\sqrt{4}\sqrt{10}$

$2\sqrt{10}$

[1]

(b) pq is equal to

$10\sqrt{40}$ $\quad$ $40\sqrt{10}$ $\quad$ 400 $\quad$ 200 $\quad$ 20

$\sqrt{40} \times \sqrt{10} = \sqrt{400}$

[1]

(c) q^5 is equal to

$100\sqrt{10}$ $\quad$ $5\sqrt{10}$ $\quad$ $\sqrt{50}$ $\quad$ 625 $\quad$ $10\sqrt{100}$

$\underbrace{\sqrt{10} \times \sqrt{10}}_{10} \times \underbrace{\sqrt{10} \times \sqrt{10}}_{10} \times \sqrt{10}$

$10 \times 10 \times \sqrt{10}$

$100\sqrt{10}$

19. *(a)* Give one example to show that the square of an irrational number is not always rationa[l]
[

$(\sqrt{2})^2 = 2$ RATIONAL X

$(\pi)^2 = \pi^2$ IRRATIONAL ✓

Number = π Square of the number = π^2

(b) Find two different irrational numbers to make the answer to the calculation below rationa[l]. Complete the calculation by filling in the three boxes.
[1

$\boxed{\sqrt{3}} \times \boxed{\sqrt{12}} = \boxed{6}$

$\sqrt{36} = 6$

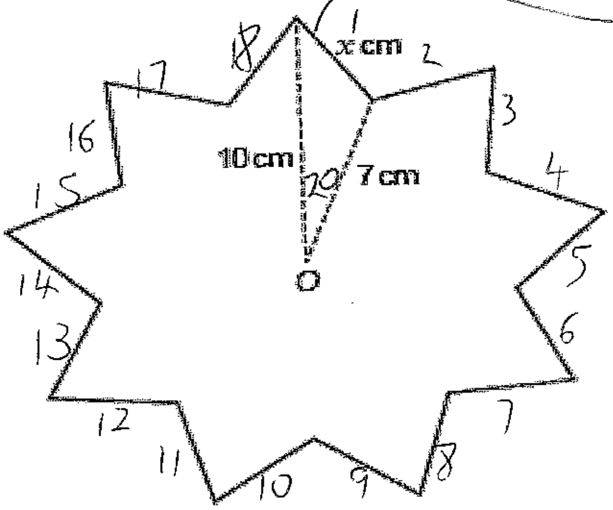
SJHS

18. A 9-pointed star, with centre O, is shown below.
Each side of the star is of length x cm.

The distance from the centre to every inner vertex of the star is 7 cm.
The distance from the centre to every outer vertex of the star is 10 cm.

~~20~~ 18 triangles

$360 \div 18 = 20°$



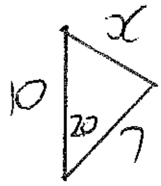


Diagram not drawn to scale

(a) Calculate the perimeter of the star. [5]

$x^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \times \cos 20$

$x^2 = 17.44$

$x = 4.18$ 18 edges

$4.18 \times 18 = 75.2$ cm

(b) Calculate the area of the star. [3]

$A = \frac{1}{2} ab \sin C$

$A = \frac{1}{2} \times 10 \times 7 \times \sin 20$

$A = 11.97$ 18 triangles

$11.97 \times 18 = 215.5$ cm^2 1dp

13.

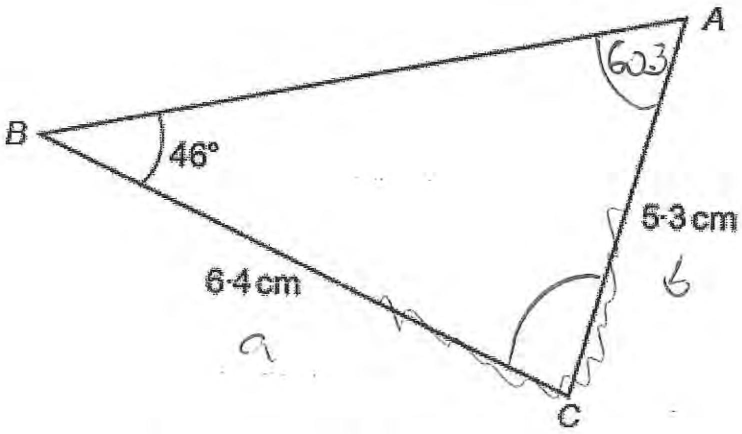


Diagram not drawn to scale

By first calculating the size of $B\hat{A}C$, calculate the area of triangle ABC.
You must show all your working.

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{6.4}{\sin A} = \frac{5.3}{\sin 46}$

$\frac{\sin A}{6.4} = \frac{\sin 46}{5.3}$

$\times 6.4 \quad \times 6.4$

$\sin A = 0.868$

$A = 60.3°$

$B\hat{C}A = 180 - 46 - 60.3$

$= 73.7$

$\text{Area} = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 6.4 \times 5.3 \times \sin 73.7$

$\text{Area} = 16.3 cm^2$

7. ABC represents the sector of a circle with radius 7 cm and centre A, as shown below.
$B\hat{A}C = x°$, $AD = 3$ cm and $BD = 6$ cm.

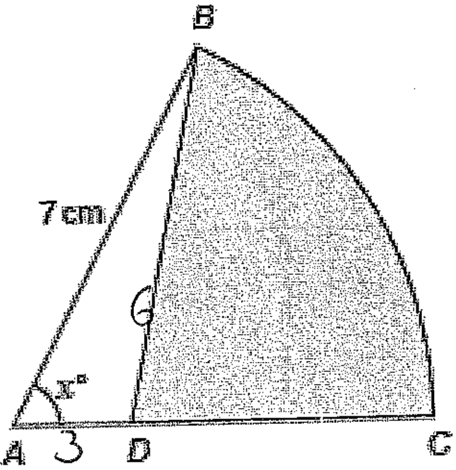
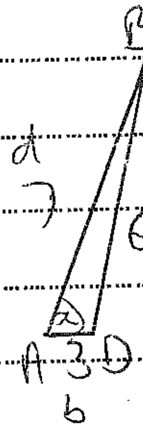


Diagram not drawn to scale

Find the area of the shaded region BCD. [8]

Area whole shape sector = Area Triangle + Grey area



$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \dfrac{3^2 + 7^2 - 6^2}{2 \times 3 \times 7}$

$\cos A = 11/21$ $\cos^{-1}$

$A = 58.4°$

Area Triangle $= \dfrac{1}{2} \times 7 \times 3 \times \sin 58.4$

$= 8.94\,cm^2$

Area Sector $= \dfrac{58.4}{360} \times \pi \times 7^2$

$= 24.97\,cm^2$

Grey area $= 24.97\,cm^2 - 8.94\,cm^2$

$= 16.03\,cm^2$ 2dp

12. Express $\frac{3x}{3x+2} - \frac{2x}{2x+7}$ as a single fraction in its simplest form. [3

$-2x$

$$\frac{3x(2x+7)}{(3x+2)(2x+7)} - \frac{2x(3x+2)}{(3x+2)(2x+7)}$$

$$\frac{6x^2+21x - 6x^2 - 4x}{(3x+2)(2x+7)}$$

$$\frac{17x}{(3x+2)(2x+7)}$$

17. Simplify $\frac{12x+16}{9x^2-16}$. [4]

$$\cancel{4(3)} \quad \frac{4(3x+4)}{(3x-4)(3x+4)}$$

$$= \frac{4}{3x-4}$$

17. Two similar shapes have areas of 700 cm^2 and 140 cm^2. The perimeter of the smaller shape is 83 cm. Calculate the perimeter of the larger shape. [3]

$ASF = \frac{700}{140} = 5$

$LSF = \sqrt{5}$

larger $= 83 \times \sqrt{5}$
$= 185.6$ cm

15. Two similar pyramids have volumes of 3970 cm^3 and 3100 cm^3 respectively. The height of the larger pyramid is 25 cm. Calculate the height of the smaller pyramid. [3]

$VSF^3 = \frac{3970}{3100} = 1.2806$

$LSF = \sqrt[3]{1.2806}$
$= 1.0859$

$25 \div 1.0859$

Height = 23.02 cm

12. *(a)* Factorise $(x-7)^2 + 2(x-7)$. **[2]**

$(x-7)[(x-7)+2]$

$(x-7)(x-5)$

(b) Factorise $12x^2 - 27y^2$. **[3]**

$3(4x^2-9y^2)$

$3(2x-3y)(2x+3y)$

16. Use the quadratic formula to solve $(3x-1)^2 = x(2x+3) + 7$.
Give your answers correct to 2 decimal places. **[6]**

$(3x-1)^2 = 9x^2-6x+1$

$x(2x+3)+7 = 2x^2+3x+7$

$9x^2-6x+1 = 2x^2+3x+7$

$\downarrow -2x^2-3x-7$

$7x^2-9x-6=0 \qquad a=7 \quad b=-9 \quad c=-6$

$x = \dfrac{9 \pm \sqrt{81-4\times7\times-6}}{2\times7}$

$x = 1.77$ or -0.48

9. (a) Show that $(10w+3)(w-1)-(2-3w)^2 \equiv w^2+5w-7$. [4]

$10w^2+3w-10w-3-(4+9w^2-12w)$

	2	$-3w$
2	4	$-6w$
$-3w$	$-6w$	$9w^2$

$10w^2-7w-3-4-9w^2+12w$

w^2+5w-7

(b) Use the quadratic formula to solve the equation $w^2+5w-7=0$. [3]
Give your answers correct to 2 decimal places.

$a=1$
$b=5$
$c=-7$

$w=\dfrac{-5\pm\sqrt{25-4\times1\times-7}}{2\times1}$

$w=1.14$ or -6.14

18. Solve the equation $x=\dfrac{7}{5x-3}$.

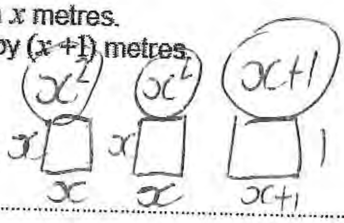
Give your answers correct to 2 decimal places. [5]

$x(5x-3)=7$
$5x^2-3x=7$
$5x^2-3x-7=0$
$a=5$
$b=-3$
$c=-7$

$x=\dfrac{3\pm\sqrt{9-4\times5\times-7}}{2\times5}$

$x=1.52$ or -0.92

14. Aled has three concrete slabs.
Two of the slabs are square, with each side of length x metres.
The third slab is rectangular and measures 1 metre by $(x+1)$ metres.
The three concrete slabs cover an area of 7 m^2.



(a) Show that $2x^2 + x - 6 = 0$. [1]

$x^2 + x^2 + x + 1 = 7$
$2x^2 + x - 6 = 0$

(b) Solve the equation to find the length of each side of the square slabs.
You must justify any decisions that you make. [4]

$(2x-3)(x+2) = 0$

$x = \frac{3}{2}$ or $x = -2$

x must be positive

so lengths are 1.5, 1.5 and 2.5 $(x+1)$

8. Factorise $x^2 - 7x - 18$, and hence solve $x^2 - 7x - 18 = 0$. [3]

$(x-9)(x+2) = 0$
$x = 9$ or $x = -2$

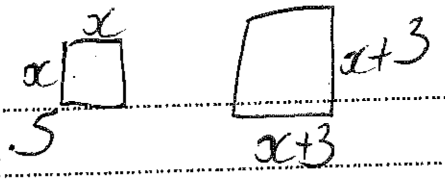
17. Simplify $\frac{12x+16}{9x^2-16}$. [4]

$$\frac{4(3x+4)}{(3x-4)(3x+4)}$$

$$\frac{4}{3x-4}$$

12. Two different squares are constructed.
The side length of the smaller square is x cm.
The side length of the larger square is 3 cm longer than the side length of the smaller square.
The combined area of the two squares is $22.5\,\text{cm}^2$.

(a) Show that $4x^2 + 12x - 27 = 0$. [4]



$x^2 + (x+3)(x+3) = 22.5$

$x^2 + x^2 + 6x + 9 = 22.5$

$2x^2 + 6x - 13.5 = 0$ $(\times 2)$

$4x^2 + 12x - 27 = 0$

(b) Find the dimensions of each of the squares.
Do not use a trial and improvement method.
You must show all your working and justify any decision that you make. [5]

$(2x + 9)(2x - 3)$

$18x$

$-6x$

$12x$

$2x + 9 = 0$ or $2x - 3 = 0$

$x = \frac{-9}{2}$ $\quad x = 1.5$

x must be positive

Side length of smaller square = 1.5 cm

Side length of larger square = 4.5 cm

16. The diagram shows two rectangles.

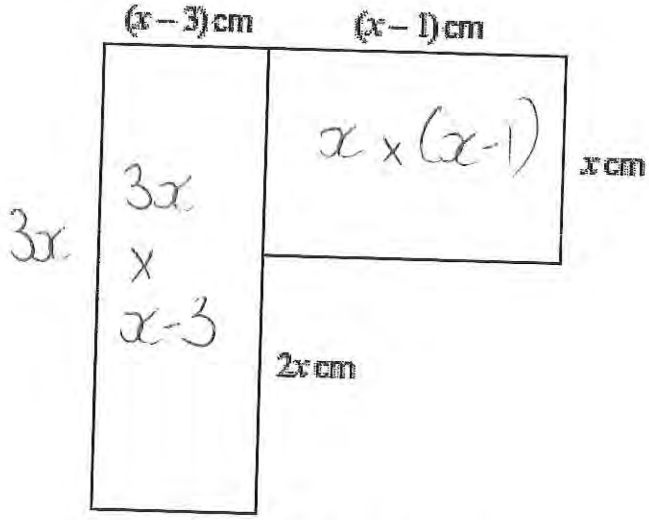


Diagram not drawn to scale

The combined area of both rectangles is 50 cm^2.
By considering the areas of the two rectangles, show that $2x^2 - 5x - 25 = 0$ and hence find the value of x.

[6]

$3x^2 - 9x + x^2 - x = 50$

$4x^2 - 10x - 50 = 0 \quad \div 2$

$2x^2 - 5x - 25 = 0$

$(2x+5)(x-5) = 0$

$x = \frac{-5}{2}$ or $x = 5$

+ must be +ve so $x = 5$

9. (a) Factorise $x^2 - 2x - 24$, and hence solve $x^2 - 2x - 24 = 0$. [3]

$(x-6)(x+4) = 0$

$x = 6$ or $x = -4$

(b) Solve the equation $\frac{4x-3}{2} + \frac{7x+1}{6} = \frac{29}{2}$. [4]

$\times 3 \quad \times 1 \quad \times 3$

$\frac{12x-9}{6} + \frac{7x+1}{6} = \frac{87}{6}$ $\quad \times 6$

$19x - 8 = 87$

$19x = 95$

$x = 5$

13. Given that y is inversely proportional to x^3 and that $y = 120$ when $x = 2$,

(a) find an expression for y in terms of x, [3]

$y \propto \frac{1}{x^3}$

$y = \frac{k}{x^3}$

$120 = \frac{k}{2^3}$

$\times 8$

$960 = k$

$y = \frac{960}{x^3}$

(b) use the expression you found in part (a) to complete the following table. [2]

x	2	10	4
y	120	0.96	15

$y = \frac{960}{10^3}$

$y = \frac{960}{1000}$

$15 = \frac{960}{x^3}$

$x^3 = \frac{960}{15}$

$x^3 = 64$

$x = 4$

11. Given that y is inversely proportional to x, and that $y = 4$ when $x = 3$,

(a) find an expression for y in terms of x, [3]

$y \propto \frac{1}{x}$

$y = \frac{k}{x}$ $\qquad$ $y = \frac{12}{x}$

$4 = \frac{k}{3}$

$k = 12$

(b) use the expression you found in (a) to complete the following table. [2]

x	3	0·25	60
y	4	48	$\frac{1}{5}$

$y = \frac{12}{1/4} \begin{matrix} \times 4 \\ \times 4 \end{matrix}$

$\frac{1}{5} \overset{\times 12}{=} \frac{12}{x}$ $\times 12$

$\frac{1}{5} = \frac{12}{60}$

17. A bag contains 6 red blocks, 4 green blocks and 2 yellow blocks. Three blocks are taken from the bag, at random, without replacement.

(a) What is the probability that the first block removed is red, the second is green and th third is yellow? [2

$\frac{6}{12} \times \frac{3}{11} \times \frac{2}{10} = \frac{36}{1320}$ or $\frac{3}{110}$

(b) Calculate the probability that all three blocks will be the same colour. [3]

RRR

$\frac{6}{12} \times \frac{5}{11} \times \frac{4}{10} = \frac{120}{1320}$

OR GGG impossible

$\frac{4}{12} \times \frac{3}{11} \times \frac{2}{10} = \frac{24}{1320}$

$= \frac{144}{1320}$ or $\frac{6}{55}$

OR YYY

IMPOSSIBLE

(c) Write down the probability that the three blocks will not be the same colour. [1]

1 - will be same

$1 - \frac{6}{55} = \frac{49}{55}$

7. 100 boxes each contain 10 balls.

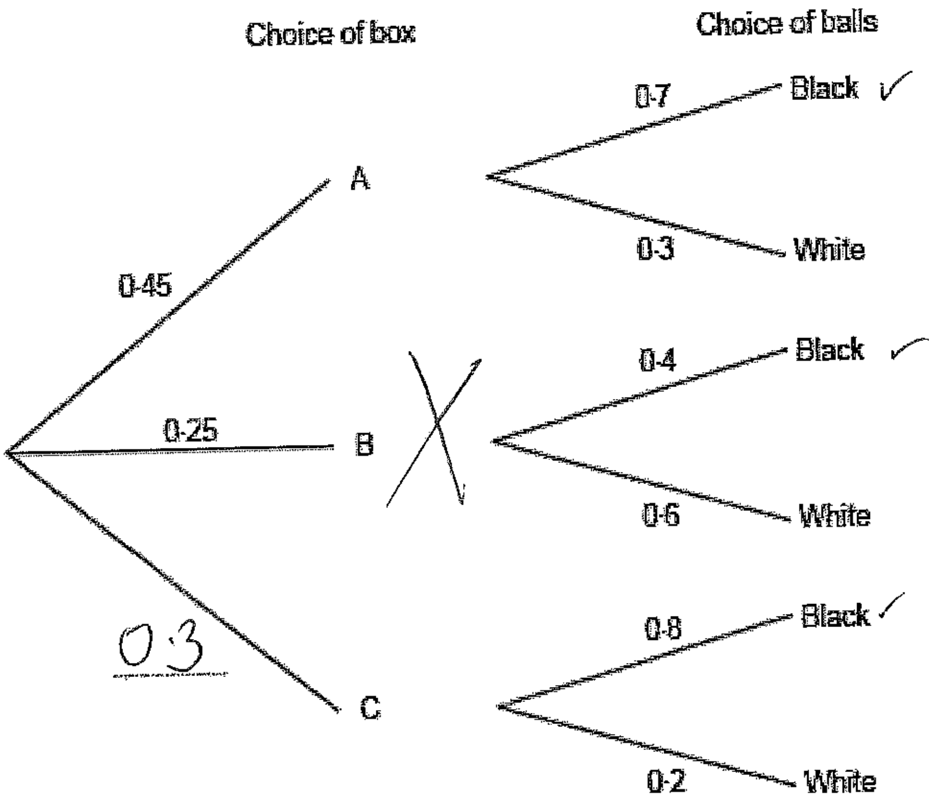
45 of the boxes are labelled A.
They each contain 7 black balls and 3 white balls.

25 of the boxes are labelled B.
They each contain 4 black balls and 6 white balls.

The rest of the boxes are labelled C.
They each contain 8 black balls and 2 white balls.

In a game, a player chooses a box at random, and then chooses a ball at random from that box.

(a) Complete the tree diagram shown below. [1]



(b) What is the probability that a player will select a black ball? [3]

$0 \cdot 45 \times 0 \cdot 7 + 0 \cdot 25 \times 0 \cdot 4 + 0 \cdot 3 \times 0 \cdot 8$

$= 0 \cdot 655$

(c) If a large number of people played the game, approximately what fraction of them would you expect to choose a white ball?
Circle your answer. [1]

$\frac{1}{10}$ $\frac{1}{5}$ $\frac{1}{4}$ $\frac{1}{3}$ $\frac{1}{2}$

$0 \cdot \dot{3}$

Black $= 0 \cdot 655$

white $= 1 - 0 \cdot 655$

$= 0 \cdot 345$

16. The table below shows the three-day rain forecast for Monday, Tuesday and Wednesday in Eglwyswrw.

Day	Probability of rain
Monday	80%
Tuesday	80%
Wednesday	80%

For these three days,

(a) calculate the probability that it will rain on all three days. [2]

Y Y Y

$0.8 \times 0.8 \times 0.8 = 0.512$

(b) calculate the probability that it will rain on exactly 2 consecutive days. [3]

YYN or NYY

$0.8 \times 0.8 \times 0.2 + 0.2 \times 0.8 \times 0.8$

$0.128 + 0.128$

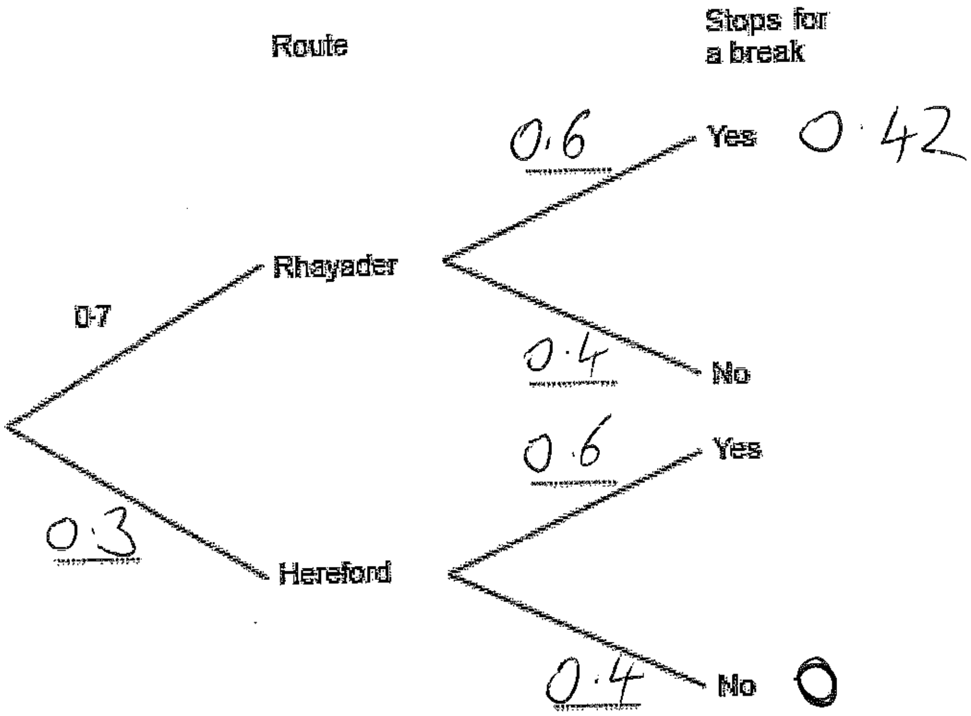
$= 0.256$

7. Alwyn often drives from Bangor to Cardiff.
He always chooses one of two routes for these journeys.
He either travels through Rhayader or through Hereford.
The probability that he travels through Rhayader is 0·7.

Sometimes he decides to stop for a break during his journey.
His decision is independent of the route he takes.

The probability that he travels through Rhayader **and** stops for a break is 0·42.

(a) Complete the following tree diagram. [4]



(b) Calculate the probability that Alwyn travels through Hereford but does not stop for a break. [2]

$0\cdot3 \times 0\cdot4 = 0\cdot12$

18. A game played at a children's party involves throwing a ball into a bucket. Each child tries to get the ball into the bucket in the least number of throws. On each attempt, the probability that Sofia gets the ball into the bucket is 0·8. Each attempt is independent of any previous attempt.

Show that she is 5 times more likely to get the ball into the bucket on her first attempt than to have her first successful throw on her second attempt.

You must show all your working. [3]

First attempt = 0.8

Second attempt = Miss and Hit
= 0.2 × 0.8
= 0.16

5 × 0.16 = 0.8

so 5 times more likely 1st attempt

13. A bag contains 5 red counters and 5 blue counters. Three counters are drawn at random from the bag at the same time. Calculate the probability that the three counters will be the same colour. [3]

RRR or BBB

$\frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} + \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8}$

$\frac{60}{720} + \frac{60}{720} = \frac{120}{720}$ or $\frac{1}{6}$

19.

2	**3**	**3**	**4**	**4**	**4**

Two of the cards shown above are selected at random, without being replaced.

Find the probability that

(a) the product of the two numbers selected is 12. [3]

$\times$

4 and 3 or 3 and 4

$\frac{3}{6} \times \frac{2}{5}$ $\qquad$ $\frac{2}{6} \times \frac{3}{5}$

$\frac{6}{30} + \frac{6}{30} = \frac{12}{30}$ or $\frac{2}{5}$

(b) the sum of the two numbers selected is even. [4]

E and E or O and O

$\frac{4}{6} \times \frac{3}{5}$ $\qquad$ $\frac{2}{6} \times \frac{1}{5}$

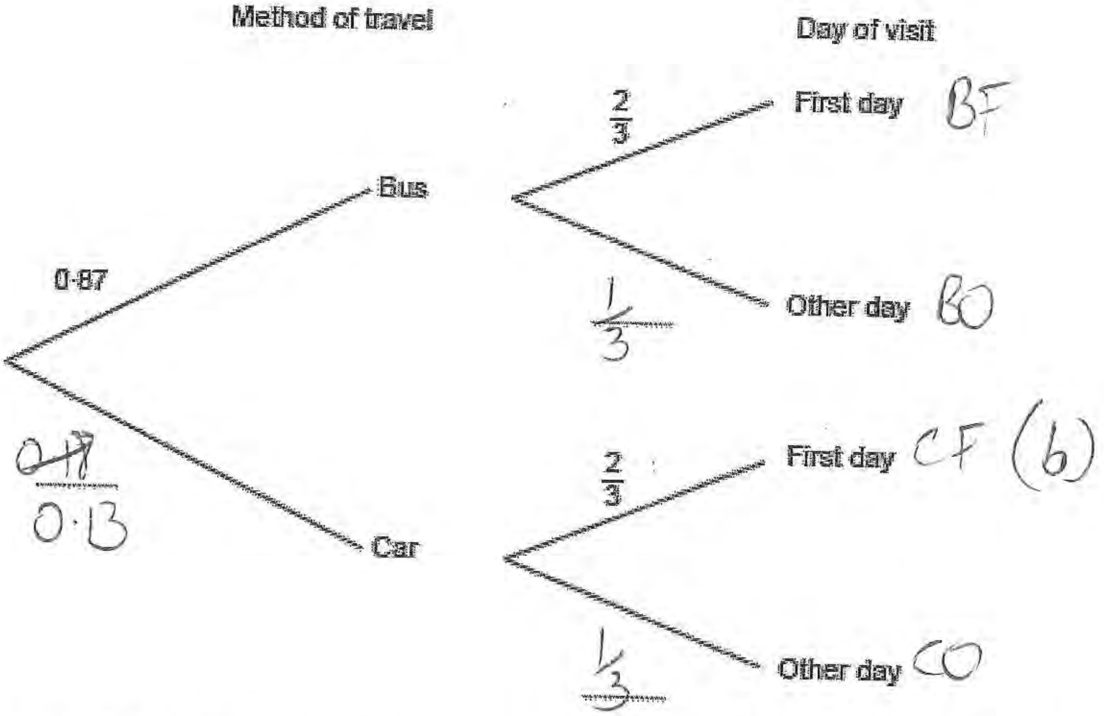
$\frac{12}{30} + \frac{2}{30} = \frac{14}{30}$ or $\frac{7}{15}$

8. All the members of a farming club visited the Royal Welsh Agricultural Show.
They all travelled to the show either by bus or by car.
None of them visited the show on more than one day.
The decision to travel by car or by bus was independent of the day of the visit.

A member of the club was selected at random.
The probability that this member travelled by bus was 0·87.

The probability that this member visited the show on the first day was $\frac{2}{3}$.

(a) Complete the tree diagram shown below. [2]



(b) What is the probability that a member, chosen at random, was not one of those who travelled by bus on the first day of the show? [3]

$0{\cdot}13 \times \frac{2}{3} = 0{\cdot}086 \quad \frac{13}{150}$

13. A bag contains 5 red counters and 5 blue counters.
Three counters are drawn at random from the bag at the same time.
Calculate the probability that the three counters will be the same colour. [3]

RRR or BBB

$\frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} \qquad \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8}$

$\frac{60}{720} + \frac{60}{720} = \frac{1}{6}$

13. Make x the subject of the following formula. **[4]**

$$a(x-b)=x(c-d)$$

$ax - ab = cx - dx$

$ax - cx + dx = ab$

$x(a - c + d) = ab$

$x = \dfrac{ab}{a-c+d}$

OR $x = \dfrac{-ab}{-a+c-d}$

12. Make c the subject of the following formula.
Give your answer in its simplest form. **[5]**

$$c-5=\frac{3c-7}{d}$$

$cd - 5d = 3c - 7$

$cd - 3c = 5d - 7$

$c(d-3) = 5d - 7$

$c = \dfrac{5d-7}{d-3}$

William has n marbles.
Lois had 4 times as many marbles as William, but she has now lost 23 of them.

Lois still has more marbles than William.

Write down an inequality in terms of n to show the above information.
Use your inequality to find the least number of marbles that William may have. [4]

W L

n $4n$

$4n - 23$

$4n - 23 > n$

$3n > 23$

$n > 7\frac{2}{3}$

$n = 8$ marbles

Rashid owned n sheep.
Eifion had exactly 4 times as many sheep as Rashid.

Rashid buys 17 extra sheep.
Eifion sells 8 of his sheep.

Eifion still has more sheep than Rashid.

Form an inequality, in terms of n.
Solve the inequality to find the least value of n.
You must show all your working. [5]

R E

n $4n$

$n+17$ $4n-8$

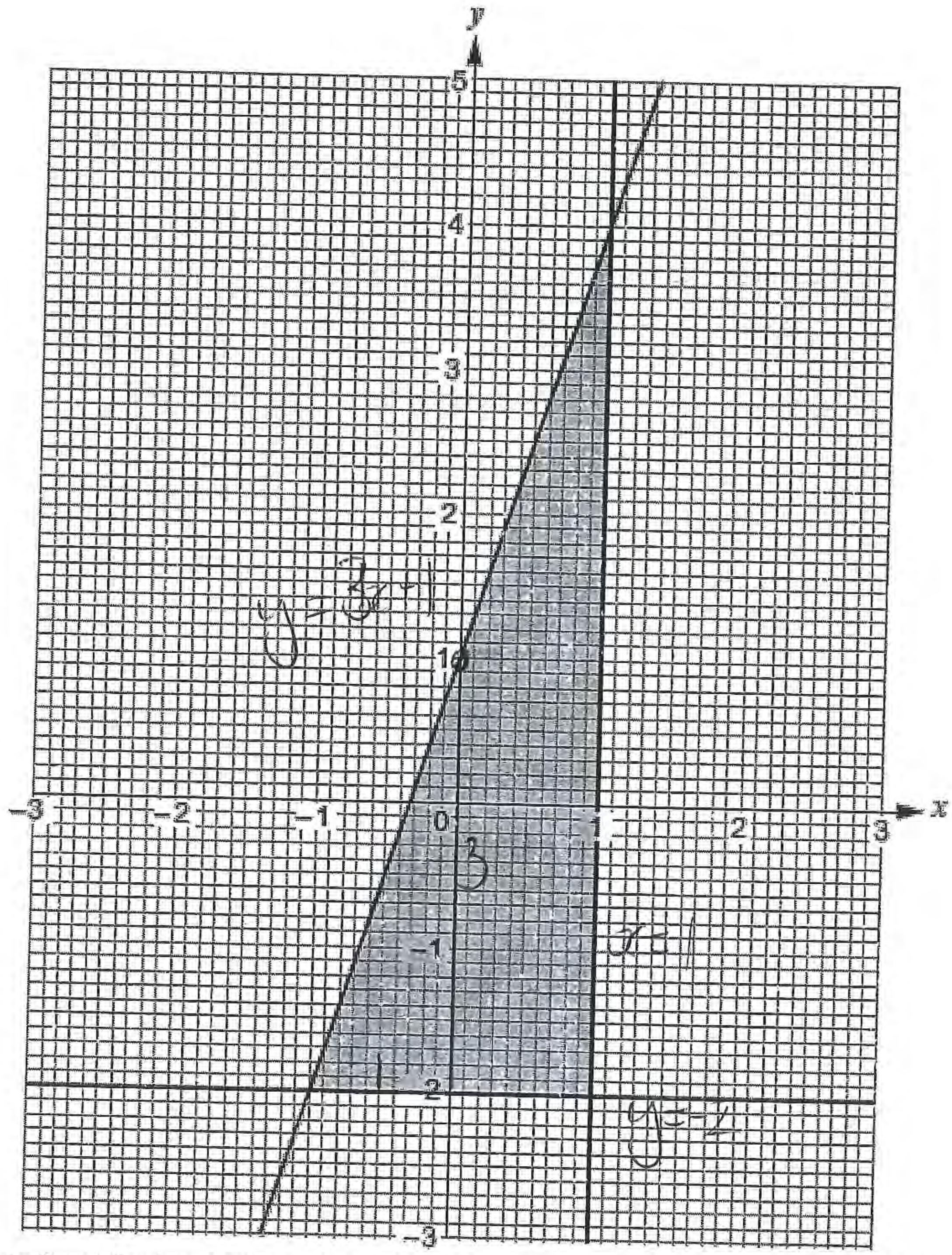
$4n-8 > n+17$

$3n > 25$

$n > 8\frac{1}{3}$

$n = 9$ sheep

11.



Complete the following table to give the set of inequalities that describes the shaded re
shown above.

....

....

$x \leqslant 1$
$y \geq -2$
$y \leq 3x+1$

13. (a) On the graph paper below, draw the region which satisfies all of the following inequalities.

$$x + y \leqslant 6 \quad ①$$
$$y \geqslant \frac{x}{2} + 3 \quad ②$$
$$x \geqslant -2.$$

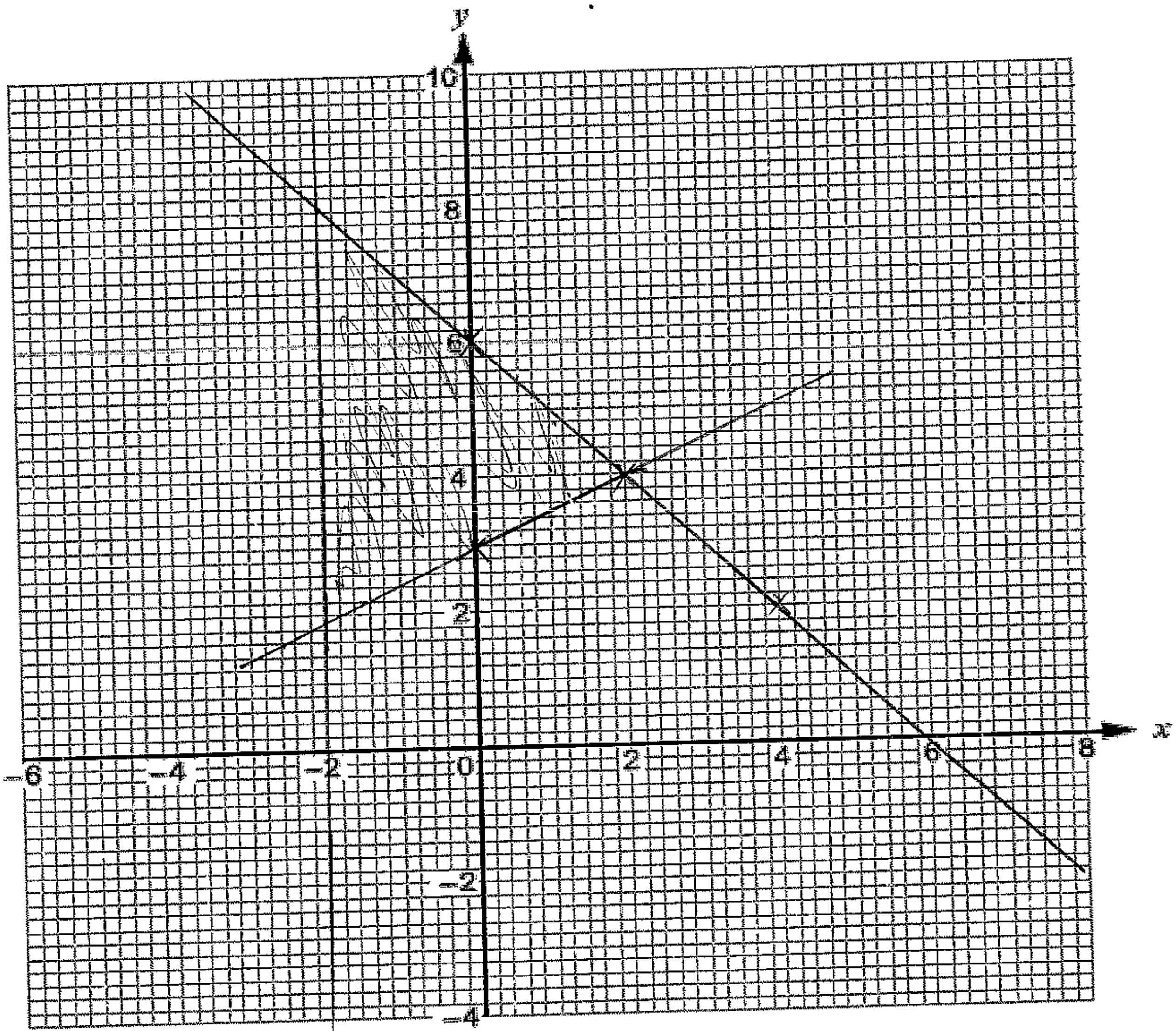
Clearly indicate the region that represents your answer. [3]

①

x	0	3	4	2
y	6	3	2	4

②

x	0	2	4
y	3	4	5



(b) (i) What is the greatest possible value of x such that all three conditions are met? [1]

$x =$ 2

(ii) What is the greatest possible value of y such that all three conditions are met? [1]

$y =$ 8

Circle the correct answer for each of the following.

(a) $x^3 \times x^6 =$ [1]

x^{36} $\quad$ $x^{0\cdot5}$ $\quad$ x^2 $\quad$ x^9 $\quad$ x^{18}

(b) $(7x - 5y) - (3x + 2y) =$ [1]

$7x - 5y - 3x - 2y$

$4x - 3y$ $\quad$ $4x - 7y$ $\quad$ $4x + 3y$ $\quad$ $-4x + 7y$ $\quad$ $-4x - 7y$

(c) A car travels x miles in 30 minutes.
Its average speed in miles per hour is [1]

$\frac{x}{2}$ $\quad$ $\frac{x}{30}$ $\quad$ $2x$ $\quad$ $\frac{2}{x}$ $\quad$ $30x$

$x = 30$

$2x = 60$ mins

Simplify each of the following and circle the correct answer in each case.

(a) $6p^6 \times 3p^3$ [1]

$9p^9$ $\quad$ $9p^{18}$ $\quad$ $18p^{18}$ $\quad$ $18p^2$ $\quad$ $18p^9$

(b) $3\cdot4g^8 \div 13\cdot6g^2$ [1]

$\frac{g^4}{4}$ $\quad$ $\frac{g^6}{4}$ $\quad$ $4g^4$ $\quad$ $4g^6$ $\quad$ $0\cdot4g^6$

(c) $\frac{m^3 \times m^6}{m^9}$ $\quad$ $\frac{m^9}{m^9} = 1 = m^0$ [1]

1 $\quad$ m $\quad$ m^2 $\quad$ m^4 $\quad$ 4

9. Circle the correct answer for each of the following statements.

(a) $9^{-\frac{1}{2}}$ is equal to

-3 $\qquad$ $-\frac{1}{3}$ $\qquad$ $\frac{1}{4\frac{1}{2}}$ $\qquad$ $-4\frac{1}{2}$ $\qquad$ $\frac{1}{3}$ $\qquad$ [1]

(b) $8^{\frac{2}{3}}$ is equal to

$5\frac{1}{3}$ $\qquad$ 4 $\qquad$ 6 $\qquad$ $8\frac{2}{3}$ $\qquad$ $\frac{16}{24}$ $\qquad$ [1]

2^2

15. (a) Express $0{\cdot}6\dot{4}\dot{2}$ as a fraction. [2]

$100x = 64.242424$

$- \quad x = 0.642424$

$99x = 63.6$

$x = \frac{63.6}{99}$

$x = \frac{636}{990}$

(b) Evaluate $\left(\frac{1}{36}\right)^{-\frac{1}{2}}$. [2]

6

17. Circle the expression that is equivalent to $w^{-\frac{3}{5}}$. [1]

$-\left(\sqrt[3]{w}\right)^5$ $\qquad$ $-\frac{3}{5}w$ $\qquad$ $-\left(\sqrt[5]{w}\right)^3$ $\qquad$ $\frac{1}{\left(\sqrt[5]{w}\right)^3}$ $\qquad$ $\frac{1}{\left(\sqrt[3]{w}\right)^5}$

11. *(a)* Evaluate $49^{-\frac{1}{2}}$. [1]

$\frac{1}{7}$

(b) Express $0{\cdot}3\dot{7}\dot{2}$ as a fraction. [2]

$100x = 37{\cdot}27272$

$- \quad x = 0{\cdot}37272$

$99x = 36{\cdot}9$

$x = \frac{36{\cdot}9}{99}$

$x = \frac{369}{990}$

11. A rectangle measures 38 cm by 26 cm.
Each measurement is correct to the nearest cm.
Calculate the least possible area of the rectangle. [2]

$37.5 \times 25.5 = 956.25cm^2$

12. The area of a rectangle is $137 cm^2$, correct to the nearest cm^2.
Its width is 11 cm, correct to the nearest cm.

Calculate the greatest possible length of the rectangle.
Give your answer correct to 3 significant figures. [2]

A 137.5 136.5

w 11.5 10.5

$\text{Big } L = \frac{\text{Big } A}{\text{small } W}$ $\frac{137.5}{10.5} = 13.1cm$

14. The region between two rectangles is shaded, as shown in the diagram below. All of the measurements shown are given correct to the nearest cm.

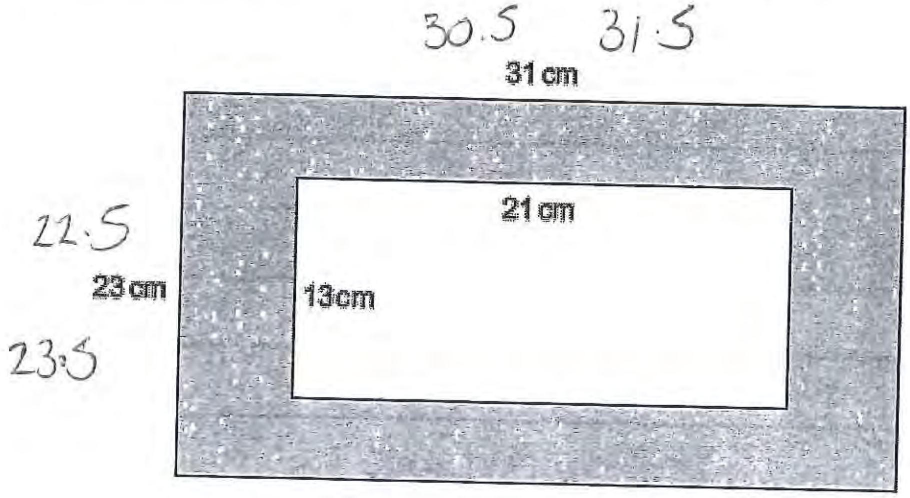


Diagram not drawn to scale

Calculate the greatest possible area of the shaded region. [3]

Grey large $31.5 \times 23.5 = 740.25$

$-$ White small $20.5 \times 12.5 = 256.25$

$= 484 \text{ cm}^2$

PQ and *PR* are tangents to a circle with centre O.
$R\hat{P}Q = 30°$.

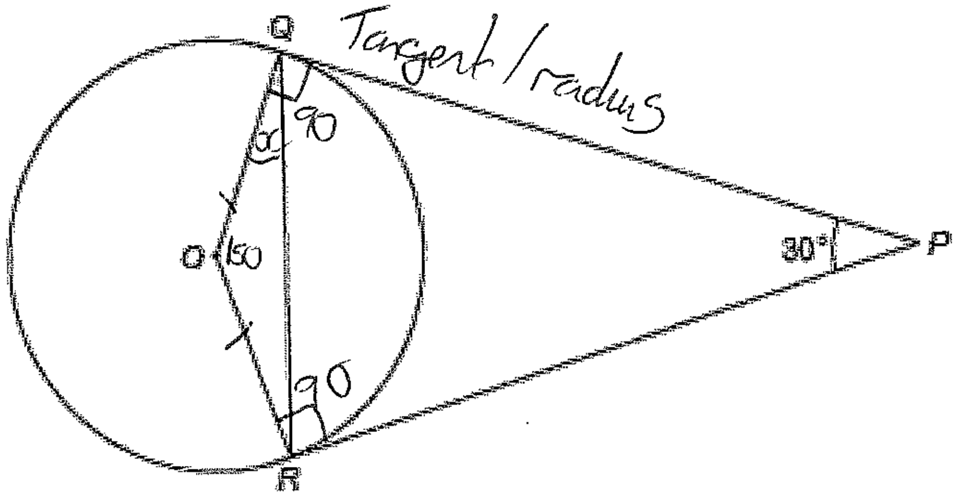


Diagram not drawn to scale

Find the size of $O\hat{Q}R$.

You must indicate any angles you calculate.
You must give a reason for each stage of your working. [5]

QOR = 360 - 90 - 90 - 30
= 150°

QOR is isosceles triangle so

$O\hat{Q}R = \frac{180-150}{2}$
$= 15°$

Points A, B, C and D lie on the circumference of a circle, centre O.
BD is a diameter of the circle.
The straight line $BC = 4{\cdot}7$ cm and $B\hat{A}C = 28°$.

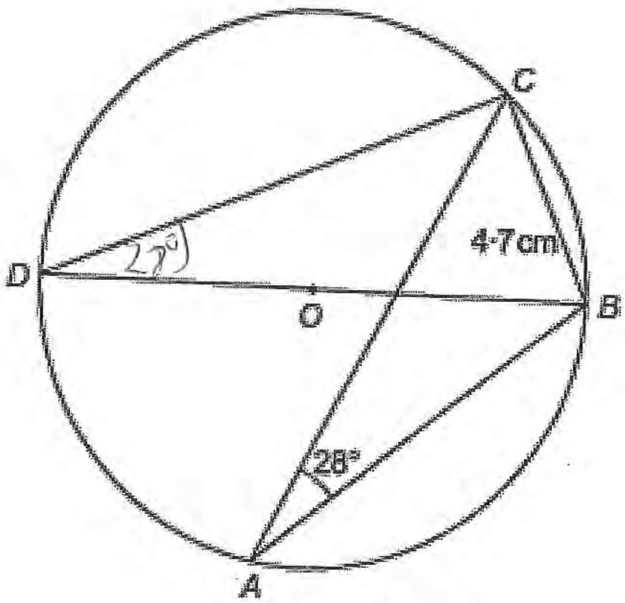
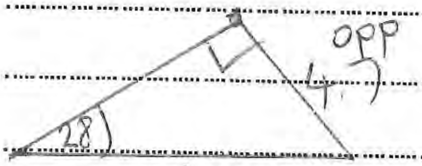


Diagram not drawn to scale

Write down the size of $B\hat{D}C$.
Hence, calculate the length BD.
You must show all your working. [5

CDB = BAD opposite same chord
ACB = 90° Opposite diameter



S O H

$x = \dfrac{4.7}{\sin 28}$

$x = 10$ cm

Points A, B and C lie on the circumference of a circle, centre O.
$A\hat{C}B = 37°$.

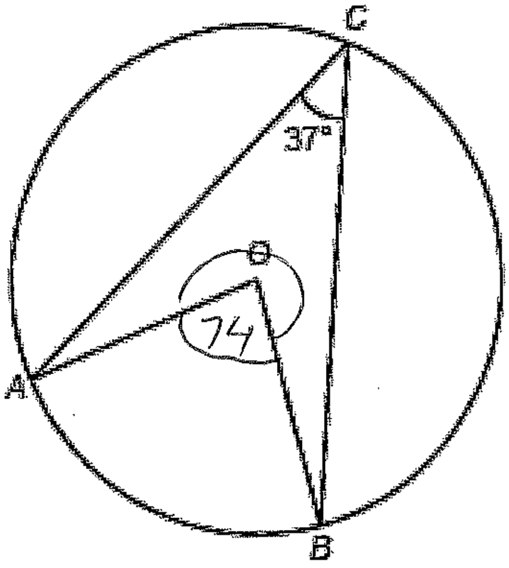


Diagram not drawn to scale

Calculate the size of the reflex angle $A\hat{O}B$. [

$360-74 = 286°$

12. A, B and C are points on the circumference of a circle.
XY is a tangent to the circle at the point A.

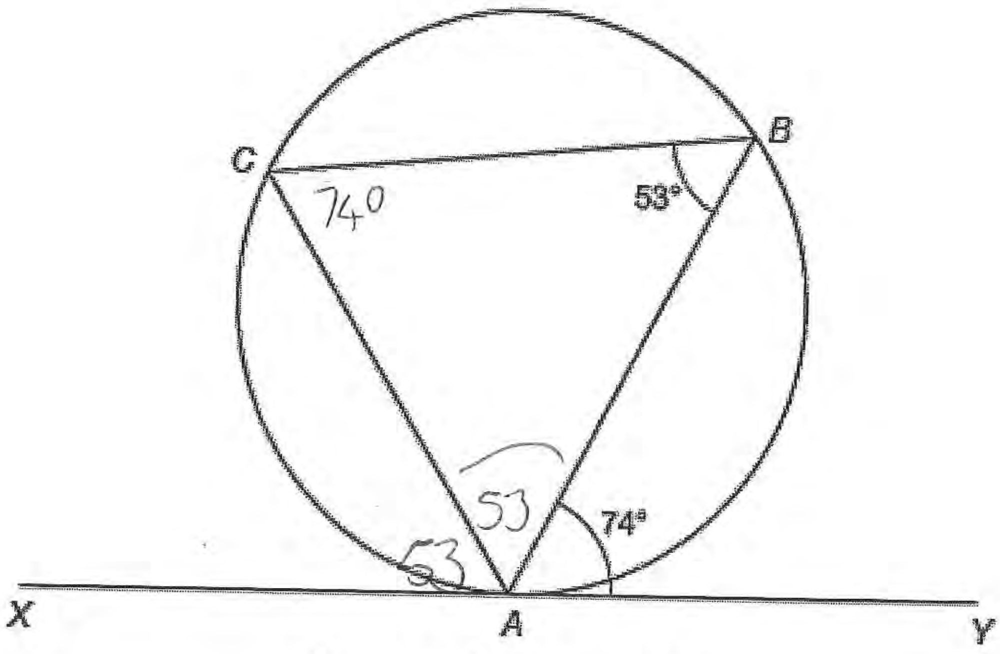


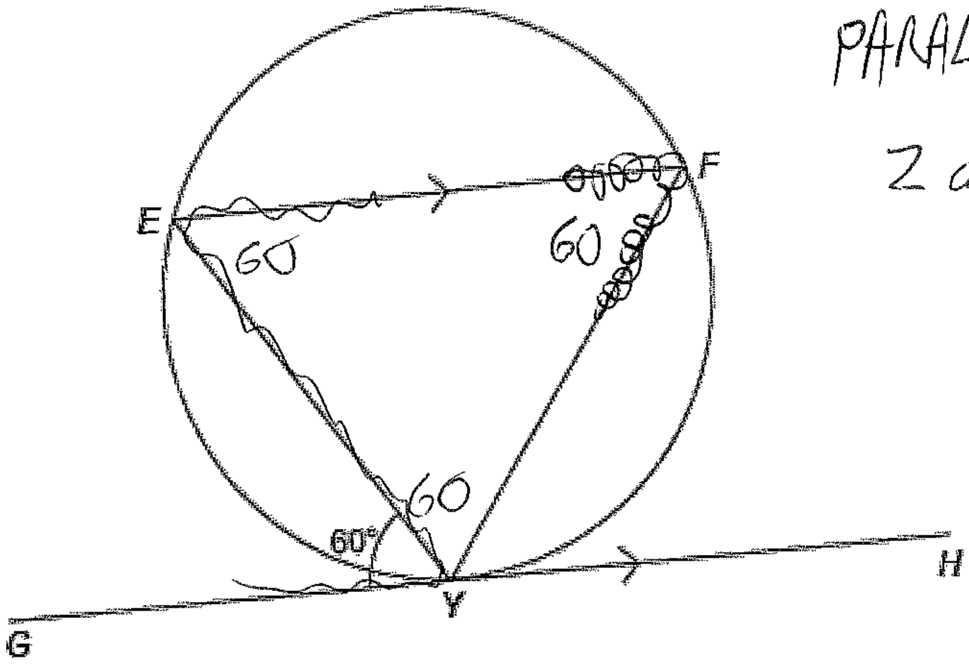
Diagram not drawn to scale

$B\hat{A}Y = 74°$ and $A\hat{B}C = 53°$.
Prove that triangle ABC is an isosceles triangle.
You must give a reason for any statement that you make or any calculation that you carry out. [5]

CAX = 53° alternate segment theorem
ACB = 74° alternate segment theorem

CAB = 180 − 74 − 53 Angles in a triangle
= 53°
Two angles the same 53° so isosceles triangle
and one different 74

10. The line GH is a tangent to the circle at point Y.
The line EF is parallel to the line GH.
The vertices of triangle EFY lie on the circle.
$E\hat{Y}G = 60°$.



PARALLEL

2 angles

Diagram not drawn to scale

Prove that EFY is an equilateral triangle.
Give a reason for each step to justify your proof. [3]

YEF = 60° alternate angles

EFZ = 60° alternate segment theorem

EYF = 180 − 60 − 60 Angles in a triangle add to 180°
= ~~180~~ 60

All angles = 60° same so equilateral triangle

13. The points P, Q and R lie on the circumference of a circle, centre O.
PQ is a diameter of the circle.
The straight line ARB is a tangent to the circle.

$Q\hat{R}B = x$, where x is measured in degrees.

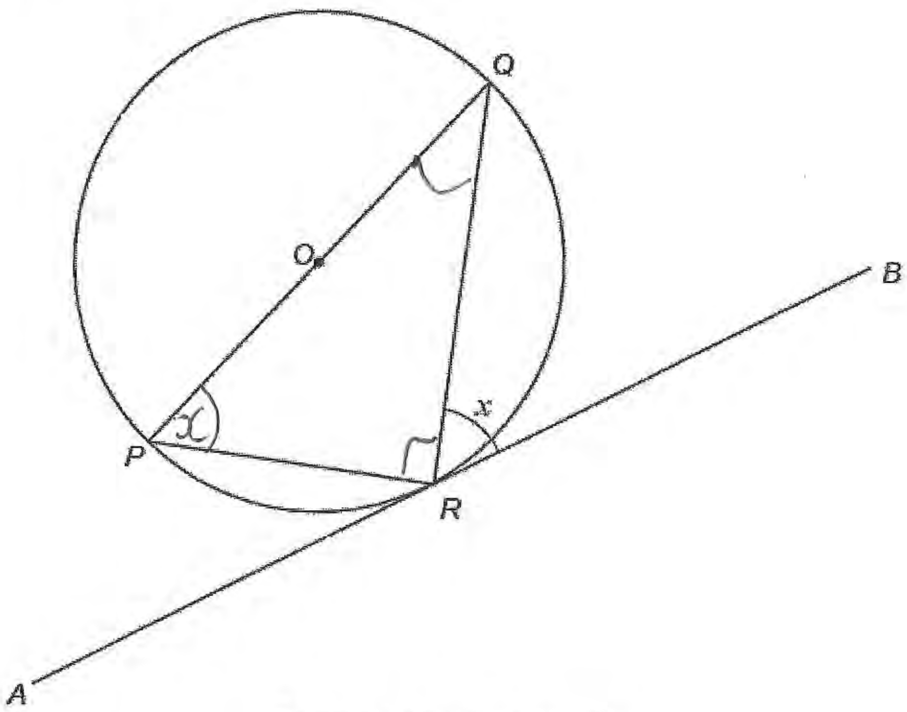


Diagram not drawn to scale

Calculate the size of $P\hat{Q}R$ in terms of x.
You must give a reason for each step of your solution. [4]

RPQ = x alternate segment theorem

PRQ = 90° Opposite diameter

PQR = 180 - 90 - x
= 90 - x

14. Points E and F lie on a circle, centre O.
The radius of the circle is 10 cm.
The area of the shaded sector is 65 cm^2.

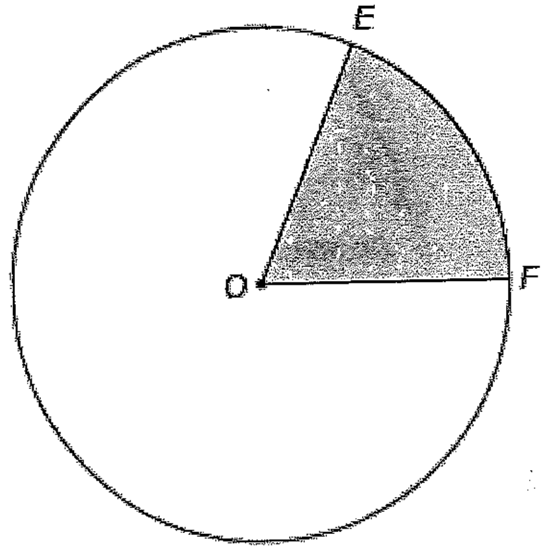


Diagram not drawn to scale

(a) Calculate the size of $E\hat{O}F$. [3]

$A = \frac{\theta}{360} \times \pi r^2$

$65 = \frac{\theta}{360} \times \pi 10^2$

$\times 360 \downarrow \times 360$

$\frac{23400}{100} = \theta \times \pi$

$\div \pi$

$\theta = 74.5^\circ$

(b) Hence, calculate the length of the arc EF. [2]

$L = \frac{\theta}{360} \times \pi D$

$L = \frac{74.5}{360} \times \pi \times 20$

$L = 13$cm

17. ABC represents the sector of a circle with radius 7 cm and centre A, as shown below. $B\hat{A}C = x°$, $AD = 3$ cm and $BD = 6$ cm.

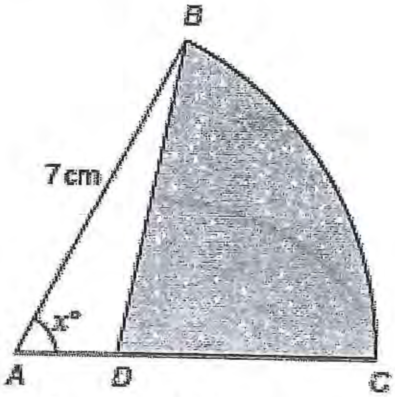
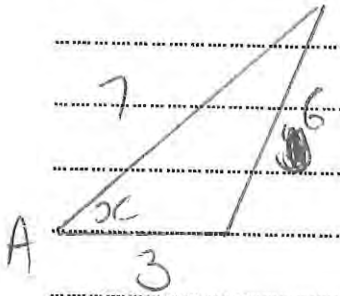


Diagram not drawn to scale

Find the area of the shaded region BCD. [8]



$\cos A = \dfrac{b^2+c^2-a^2}{2bc}$

$\cos A = \dfrac{3^2+7^2-6^2}{2\times 3\times 7}$

$\cos A = 11/21$

$A = 58.4°$

Area of triangle $= \frac{1}{2} \times 7 \times 3 \times \sin 58.4$

$= 8.94 cm^2$

Area sector $= \dfrac{58.4}{360} \times \pi \times 7^2$

$= 24.97$

Grey area $= 24.97 - 8.94$

$= 16.03 cm^2$

16. Triangle ABC is an isosceles triangle with $A\hat{B}C = A\hat{C}B$.

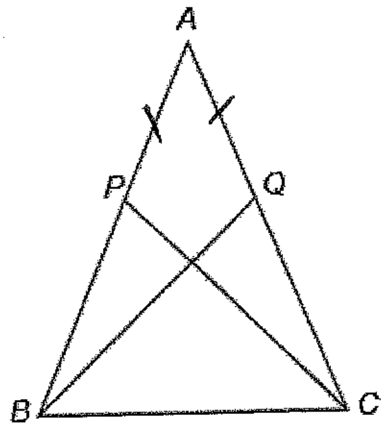


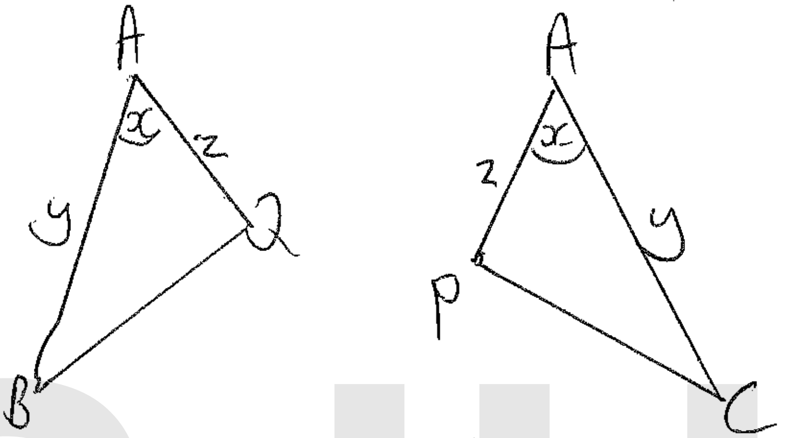
Diagram not drawn to scale

P and Q are points on AB and AC respectively such that $AP = AQ$.

Prove that triangle ABQ is congruent to triangle ACP.
You must give reasons for each step of your proof. [4]

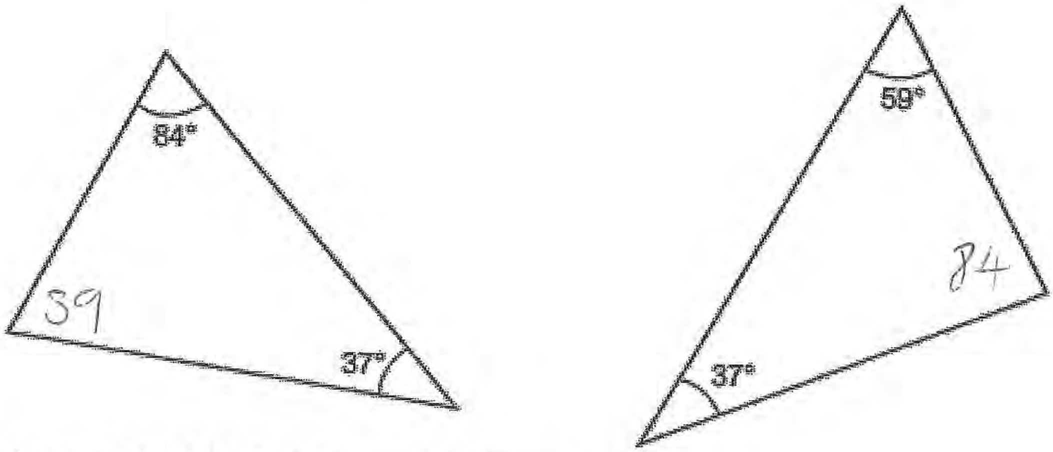
AP = AQ so PAQ is an isoscoles angle
making A the same in BAQ as CAP = x

As ABC is isosceles AB = AC = y
AP = AQ = z



So triangles are congruent
by side/angle/side being the same
SAS

14. The two triangles shown below are not drawn to scale.



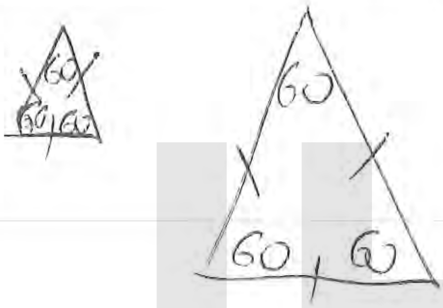
Which one of the following statements is correct?
Give full reasons for your answer. [2]

A: the triangles must be congruent
B: the triangles could be congruent
C: the triangles cannot be congruent

The correct statement is B

This is because AAA. All angles are the same but this implies similarity not congruence. They could be identical if all lengths are the same too

e.g.



All angles are same but not congruent

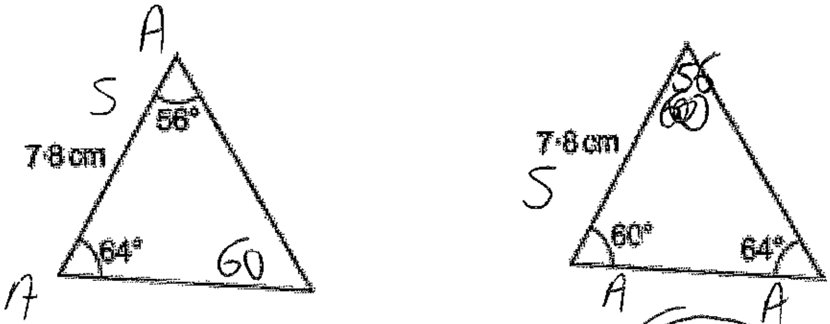
14. SSS, SAS, ASA and RHS are notations used to describe the conditions required to prove that two triangles are congruent.
[S ≡ Side, A ≡ Angle, R ≡ Right angle and H ≡ Hypotenuse.]

The following triangles are not drawn to scale.
For each pair of triangles, circle the correct statement.

(a) [1]

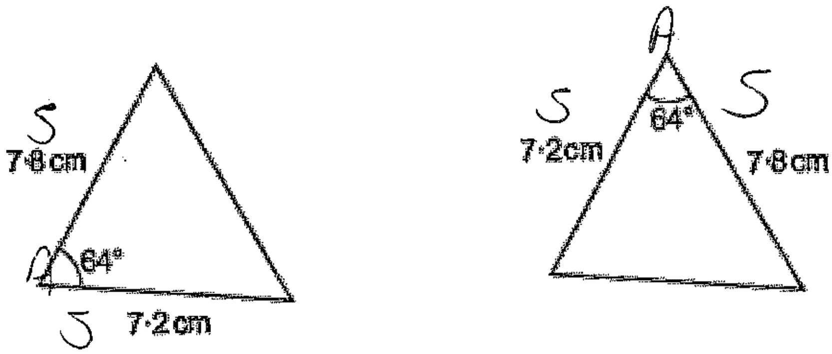
congruent: SSS | congruent: SAS | congruent: ASA | congruent: RHS | definitely not congruent | not necessarily congruent

(b) [1]



congruent: SSS | congruent: SAS | congruent: ASA | congruent: RHS | definitely not congruent | not necessarily congruent

(c) [1]



congruent: SSS | congruent: SAS | congruent: ASA | congruent: RHS | definitely not congruent | not necessarily congruent

21. The cube below has an internal diagonal of length 20 cm.
Each edge of the cube is of length x cm.

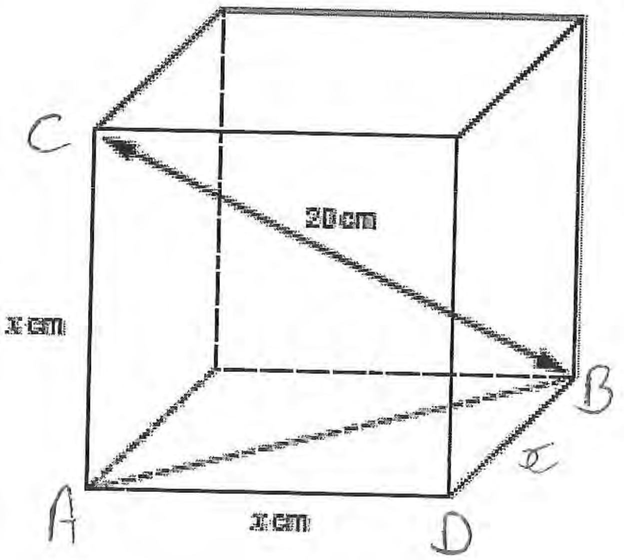
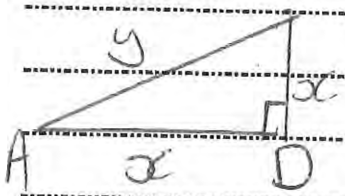
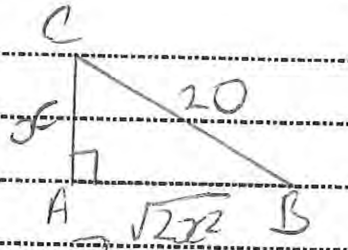


Diagram not drawn to scale

Calculate the value of x.
You must use an algebraic method and show all your working.

[4]





$y^2 = x^2 + x^2$

$y = \sqrt{2x^2}$

$20^2 = x^2 + (\sqrt{2x^2})^2$

$400 = x^2 + 2x^2$

$400 = 3x^2$

$\frac{400}{3} = x^2$

$x = \sqrt{\frac{400}{3}}$

$x = 11.55$cm 2dp

In the following formulae, each measurement of length is represented by a letter.

Consider the dimensions implied by the formulae.
Write down, for each case, whether the formula could be for a length, an area, a volume or none of these.

The first one has been done for you. [3]

Formula	Formula could be for
$d^3 - 3{\cdot}14r^2h$	volume
$d^2 + hw$ 2 2	Area
$d + w + h$ / / /	Length
$2\pi r - \pi r^2$ / 2	None
$(d + h)w$ dw +hw 2 2	Area
$d^3 + dwh$ 3 3	Volume